Articulatory-Acoustic Relations

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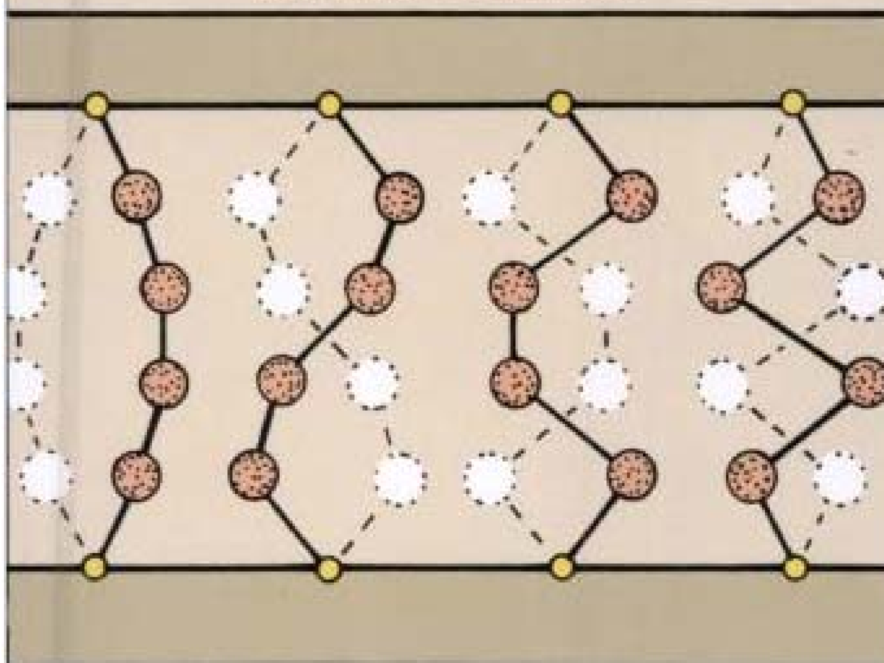
Causal Link Art -> Ac

- The goal of this presentation is to intuitively derive why /a/ has F1 and F2 close and /i/ has F1 and F2 far.
- How do changes in the articulatory **system** change acoustic parameters of the **signal**?
- Air vibration is hard to visualize, so we will start with a simple system analogous to air vibration.

FUNDAMENTALS OF MUSICAL ACOUSTICS

Second, Revised Edition



Arthur H. Benade

Vibration Essentials

- You need a physical system that impedes acceleration (mass: $F = ma$) combined with one that impedes displacement (spring $F = kx$).
- Impedance to displacement from neutral forces system back to neutral.
- A mass in motion, stays in motion, i.e. does not impede constant velocity. It will impede being accelerated or decelerated.

Steps in Vibration

- Pull mass, stretching spring away from neutral position.
- Spring resists displacement and generate force $F = kx$ to bring it back to neutral and starts moving to neutral.
- At neutral x, mass has a velocity. Stopping there (what the spring would like) could only happen if deceleration happens . Which is impeded by mass, so mass keeps moving with force $F = ma$.
- That compresses the spring eventually stopping the motion.
- The spring does not like being compressed either and a force $K = -kx$ stretches it back to neutral position.
- Mass keeps on moving, system stretches again, etc. etc. forever.

Frequency

- The number of times that the mass goes back and forth is its frequency of vibration, main property of signal we are interested in.
- <u>Perturbation Theory</u>: If you perturb a physical property of the system essential for vibration (m or k), how does that affect the frequency of the signal.
  - how does the frequency change if the mass increases?
  - How does the frequency change if the stiffness of the spring increases?

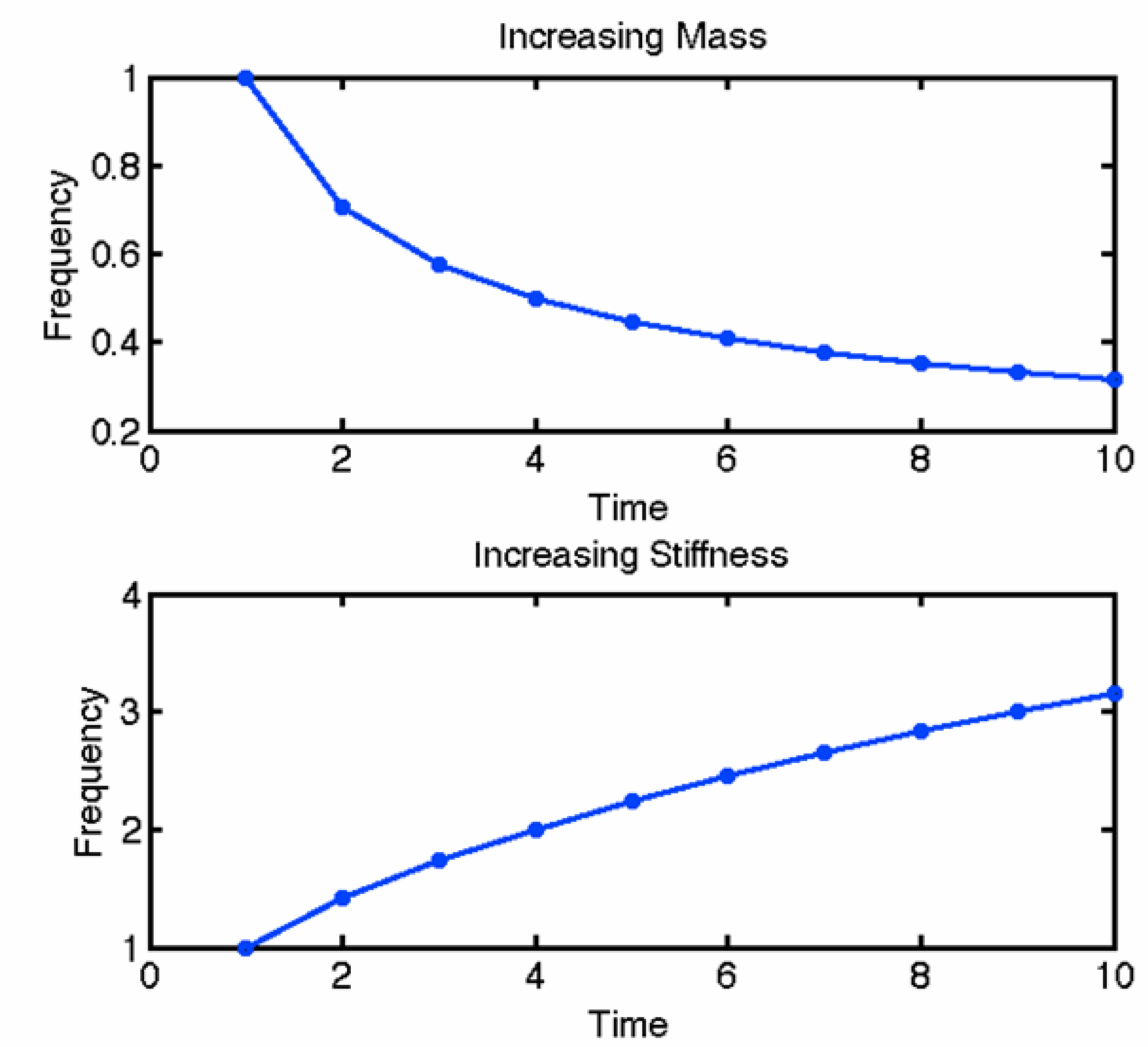
Principle 1:
Perturbation Theory for Mass Spring

- If you increase the mass, you lower the frequency, because a heavy system accelerates less.
- If you increase the stiffness, you increase the frequency, because a stiff system moves back with greater force.

Perturbation in time

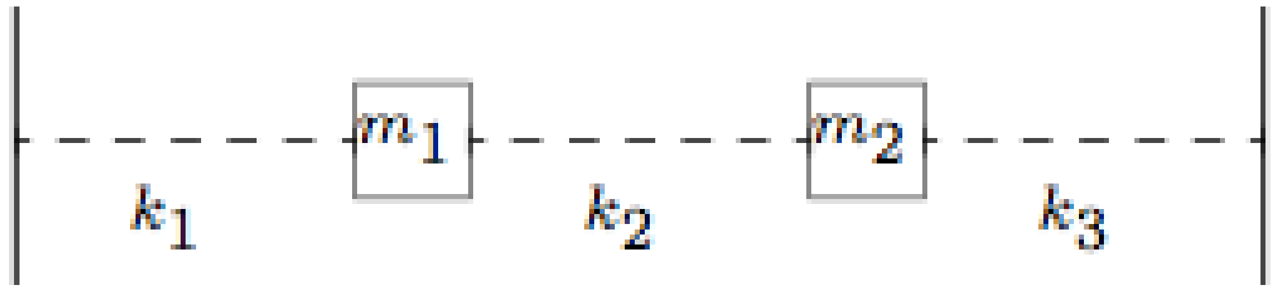
- As a mass-spring system vibrates, let's say you could somehow increase the mass as the system vibrates, as a function of time. Or let's say you could increase the stiffness as a function of time. Here are the spectrograms you would get for this time-varying system.
- The actual formula for frequency: f = $\sqrt{\frac{k}{m}}$

Spectrogram of one mass system



2-Mass 3-Spring System

- Wall-Spring-Mass-Spring-Mass-Spring-Wall



Pull both masses up. The system will vibrate by both masses going up and down in phase.

Now pull one mass up and one mass down. The two masses will vibrate out of phase.

So

- In a vibratory system with 2 m and 3 k, there will be **2 modes of vibration**.
- What will be the frequencies of vibration?
- <u>In-phase mode</u>: the middle spring just rides up and down with the masses.
- <u>Out-of-phase mode</u>: the middle spring stretches and compresses.
- Therefore: OP Mode has more effective stiffness **(3 springs vs. 2)** and therefore has higher frequency.

Summary

- Principle 1: PT for 1-Mass 1-Spring is
  - m$\uparrow$f$\downarrow$
  - k$\uparrow$f$\uparrow$
- Principle 2: A vibratory system with more than one mass can have more than one mode of vibration, *and the higher modes have higher frequencies of vibration* .

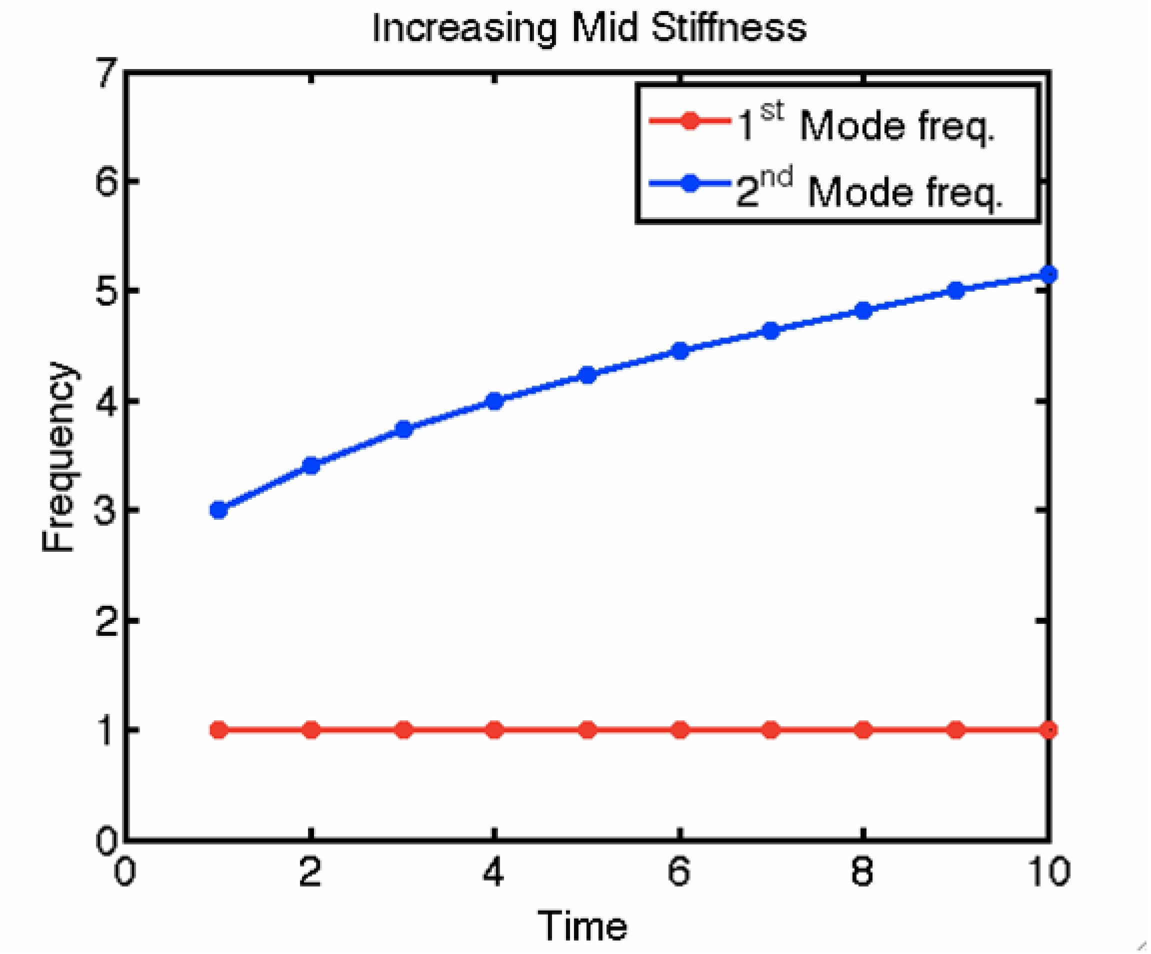
Perturb 2-mass, 3 spring

- Now increase the mass of either mass as the system vibrates. What will happen to both frequencies?
- They will both gradually drop, since the masses move in both modes, therefore making each slower to return to neutral.
- Increase stiffness of two peripheral springs, and both frequencies will increase.

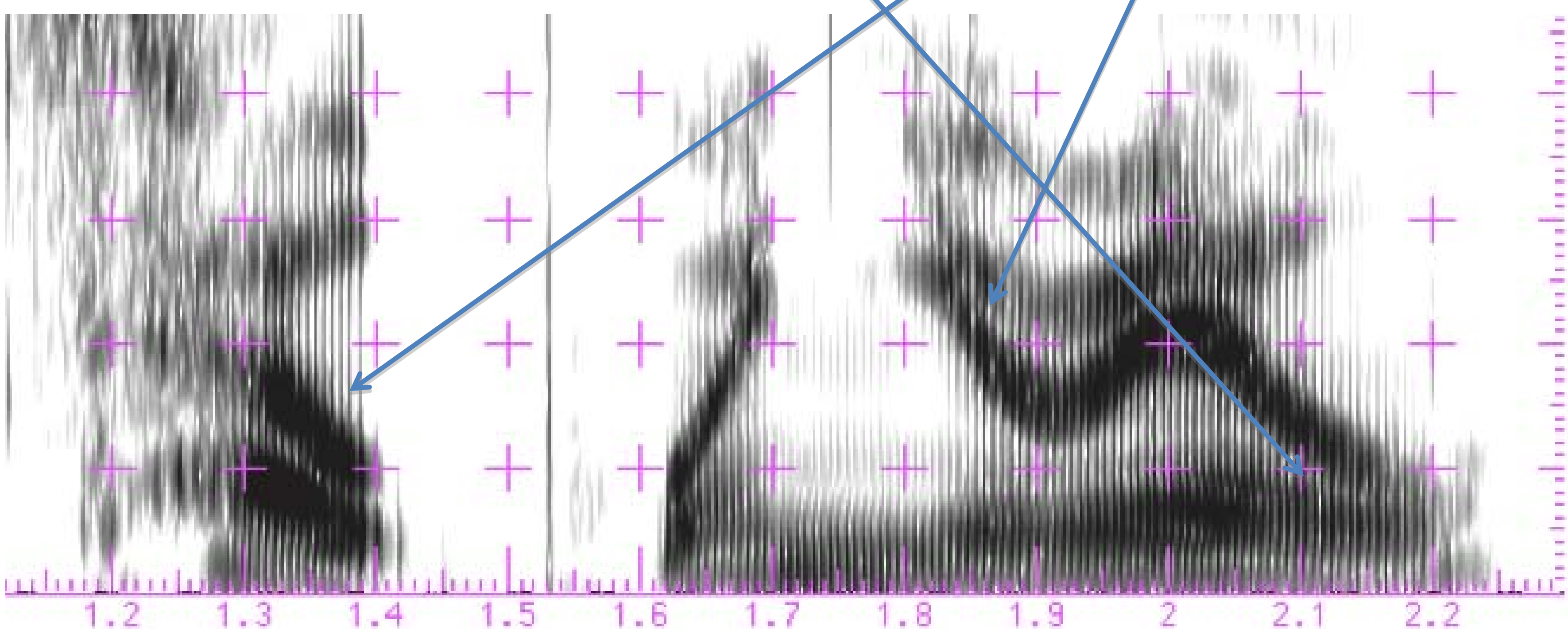
But…

- Now increase the stiffness between the masses. What will happen to the frequencies of both modes?
- The second mode frequency will increase, since it uses the middle spring.
- But in the first mode, the middle spring does not move at all, so why should increasing its stiffness affect the frequency? It doesn’t!
- So the first mode frequency will be stationary, as the second mode frequency drops!

Spectrogram of two mode system, as mid stiffness increases



Now, you can kind of understand why formants can be synchronous asynchronous



Summary

- <u>Principle 1</u>: PT for 1-Mass 1-Spring is
  - m↑f↓
  - k↑f↑
- <u>Principle 2:</u> In a multiple mass-spring system, there will be multiple modes of vibration, each successive one using more effective stiffness and hence higher frequency.
- <u>Principle 3</u> is Formant Asynchrony: Increasing/decreasing m or k, will have different effects on different frequencies, depending on whether that m or k is used or not (or used very little). Effect of m or k decrease is <u>a function of position</u>.

Principle 3 -> /a/ vs. /i/

- Principle 3 is very important since it shows that perturbing a mass/spring in one location will have a totally different effect on frequencies of vibration than making the same perturbation elsewhere.
- As you will see, /a/ and /i/ involve the same perturbation, but different locations, leading to different formant patterns.

3-mass 4-spring, 2 rigid walls

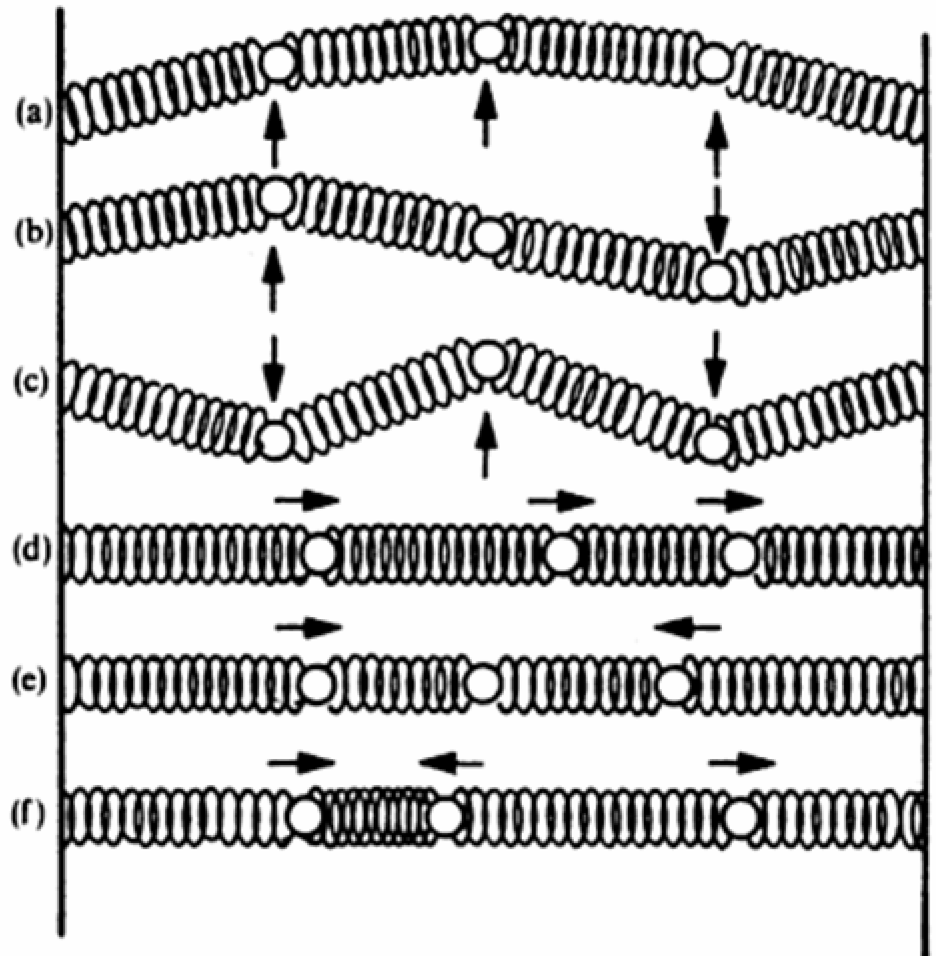


FIGURE 2.1. Normal modes of a three-mass oscillator. Transverse mode (a) has the lowest frequency and longitudinal mode (f) the highest.

N-mass -> N-Mode

- You can see the generalization emerging:
- For each additional mass, there is an additional mode of vibration.
- The previous principles remain fully valid.

∞Mass-Spring = String

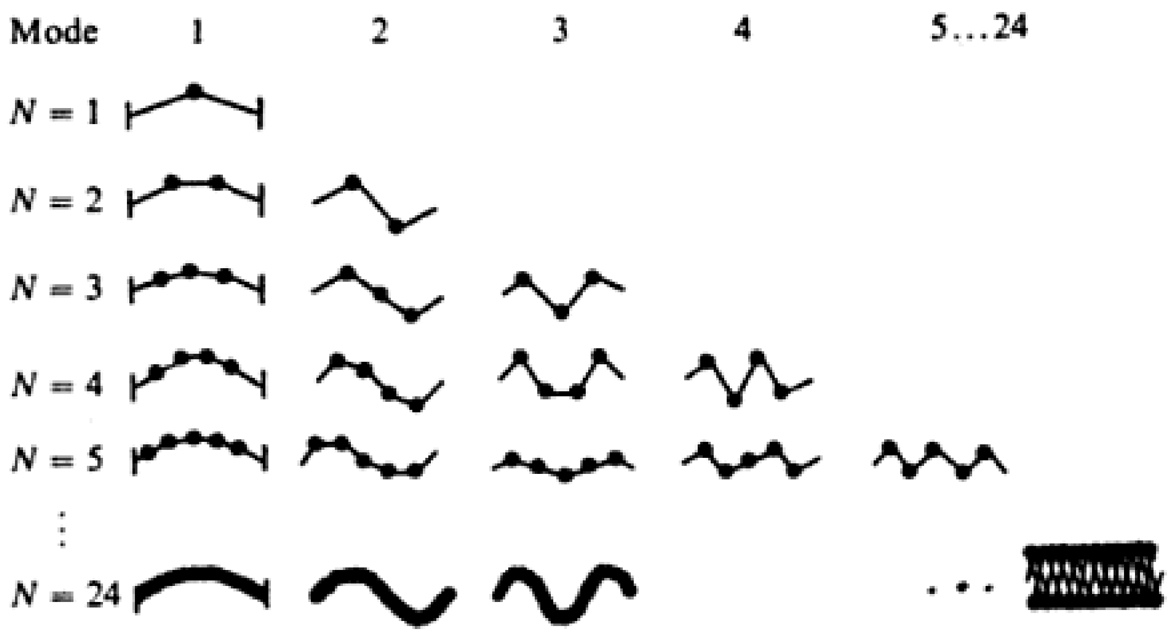
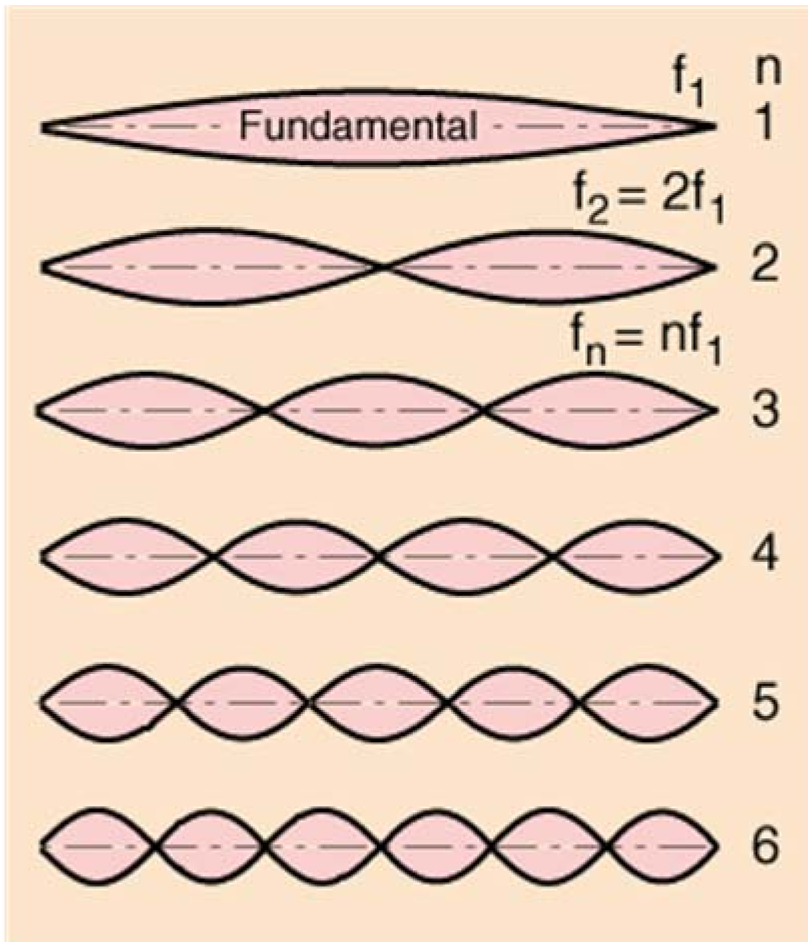


FIGURE 2.2. Modes of transverse vibration for mass/spring systems with different numbers of masses. A system with N masses has N modes.

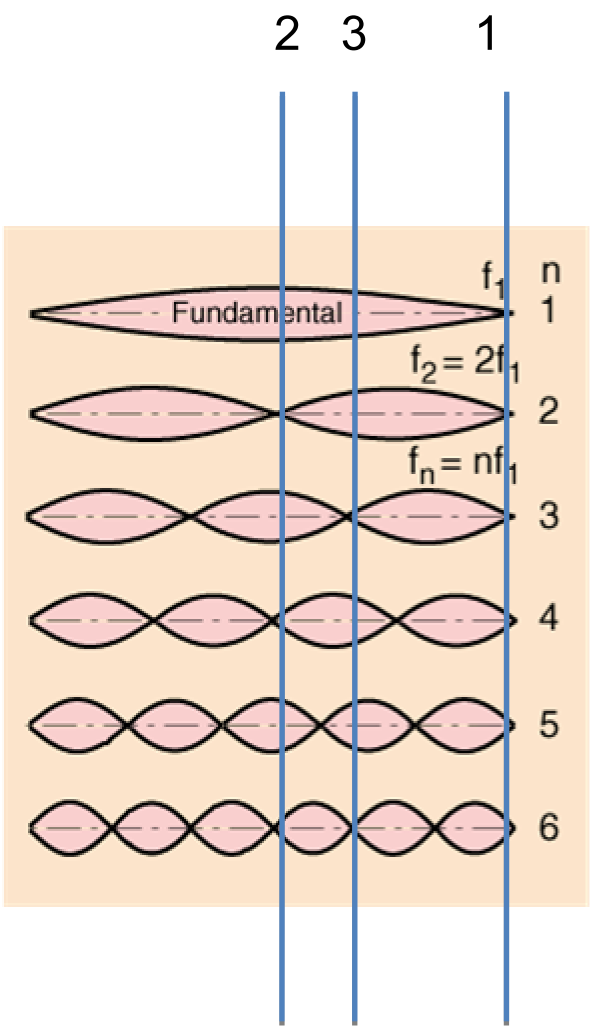
Lagrange: ∞ # of m and k + 2 Walls = Wall+String+Wall ∞ # of modes



- **Boundaries**: Wall or no wall
- Walls fix motion to 0. No-walls fix stretchiness to 0.
- An increase in stiffness near a wall has maximal increase effect on the frequency, since stretchiness is maximal there.
- An increase of mass near a no-wall has maximal lowering effect on frequency.
- Distance from a wall will determines the degree of effect of an m or k change on frequency.
- Close to wall, mass ineffective. Close to no-wall, stiffness ineffective.

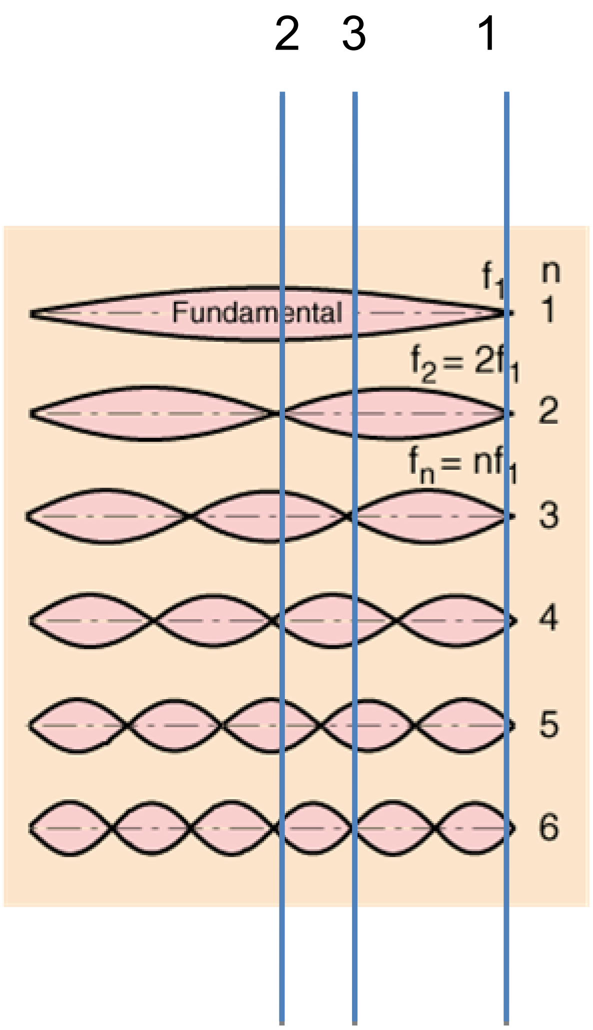
- <u>1. Ends</u>: No effect on frequencies, since there is no motion, and increased effect of mass is through motion.
- <u>2. Middle</u>: Odd Modes will decrease and even ones will not be affected, since mass is fixed there.
- <u>3. One third</u>: M12 down, M3, No change, M4 small change.M5, slightly higher change. M6, no change…

Effect of Mass Change on frequency, by location

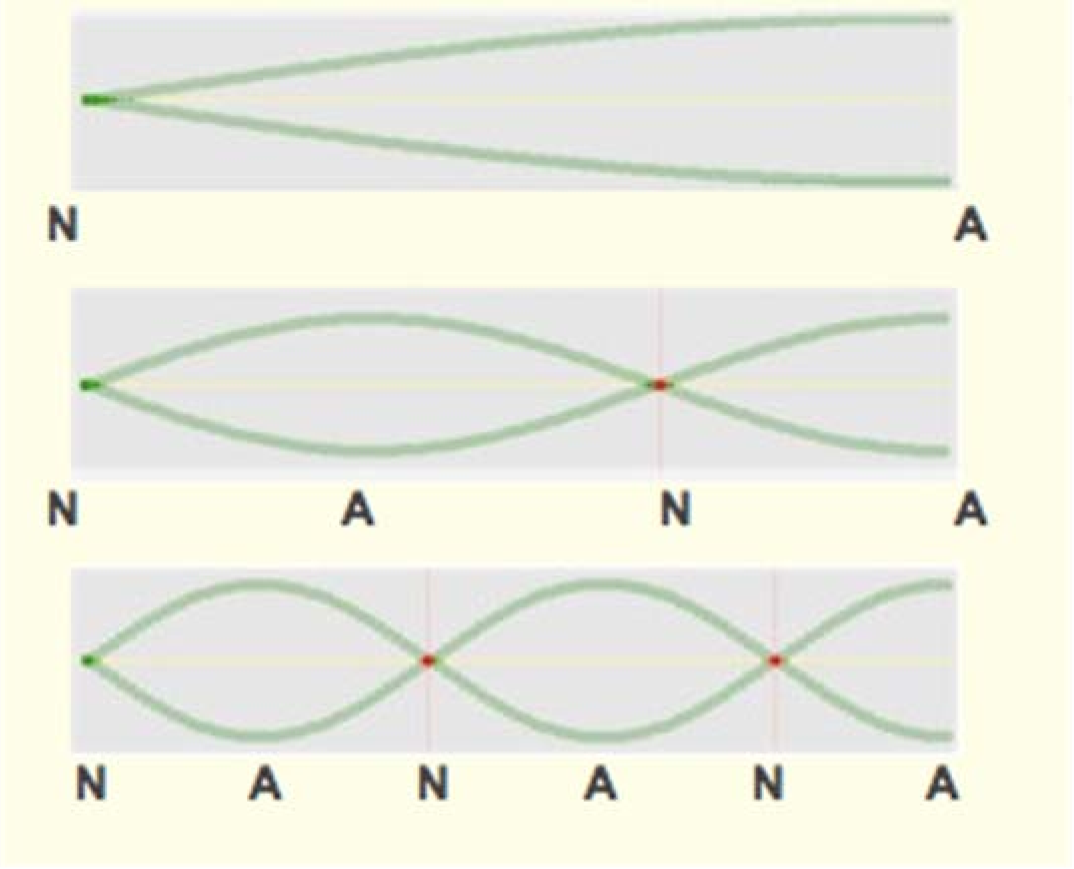


Effect of k Change on frequency, by location

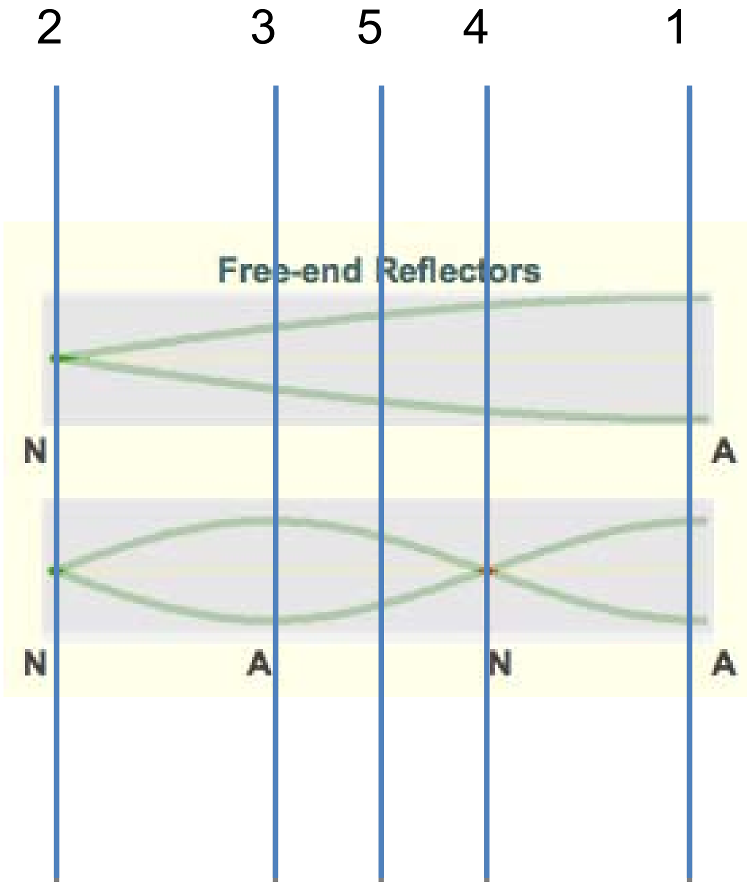
- <u>1. Ends</u>: Max + Effect, since there is max effective k.
- <u>2. Middle</u>: No effect on odd modes. Maximal + effect on even moved, since they maximally stretch at the center.
- <u>3. One third</u>: M12 up, M3, Max change, M4 less change. M5, near 0. M6, Max…



Wall+String+NoWall



	m↑	k↑	m↑k↑
1) Open End	All F_i decrease	No Effect	All F_i decrease
2) Closed End	No Effect	All F_i increase	All F_i increase
3) 1/3	F_1: Small ↓ F_2: Max ↓	$F_{1:}$: Lrge ↑ F_2: No Eff.	$F_{1:}$: Mid ↑ F_2: Max ↓
4) 2/3	F_1: Large ↓ F_2: No Eff.	F_1: Small ↓ F_2: Max ↑	F_1: Large ↓ F_2: Max ↑
5) Mid	F_1: Small ↓ F_2: Small ↓	F_1: Small ↑ F_2: Small ↑	F_1: No Eff. F_2: No Eff.



Constriction: m↑k↑

- Portions of air have mass and springiness.
- Constricting a portion of air by constricting a tube:
  - Raises the mass, since packed molecules are harder to move, i.e. a constriction raises density.
  - Raises the stiffness (as in a tire), i.e., a constriction raises pressure.
- So a constriction in a tube amounts to raising both mass and stiffness.

Putting it all together

- Closed-Open vocal tract is analogous to fixed-free string.
- Cons. at the lips /u/ = reduce all formants
- Cons. at the glottis = raise all formants
- Cons. at 1/3 VT /a/: F1 up and F2 down
- Cons. at 2/3 VT /i/: F1 down and F2 up

Adding a little Math

- Now we want to actually calculate how much each formant frequency changes as a constriction is introduced.
- The simplest perturbations are perturbations of a neutral schwa-like tube. What do we need for the math?
  - Mode Frequencies for schwa
  - Function representing the perturbation
  - Function representing how sensitive each mode frequency is to a perturbation at each location in tube

A) CO Neutral Mode Frequencies

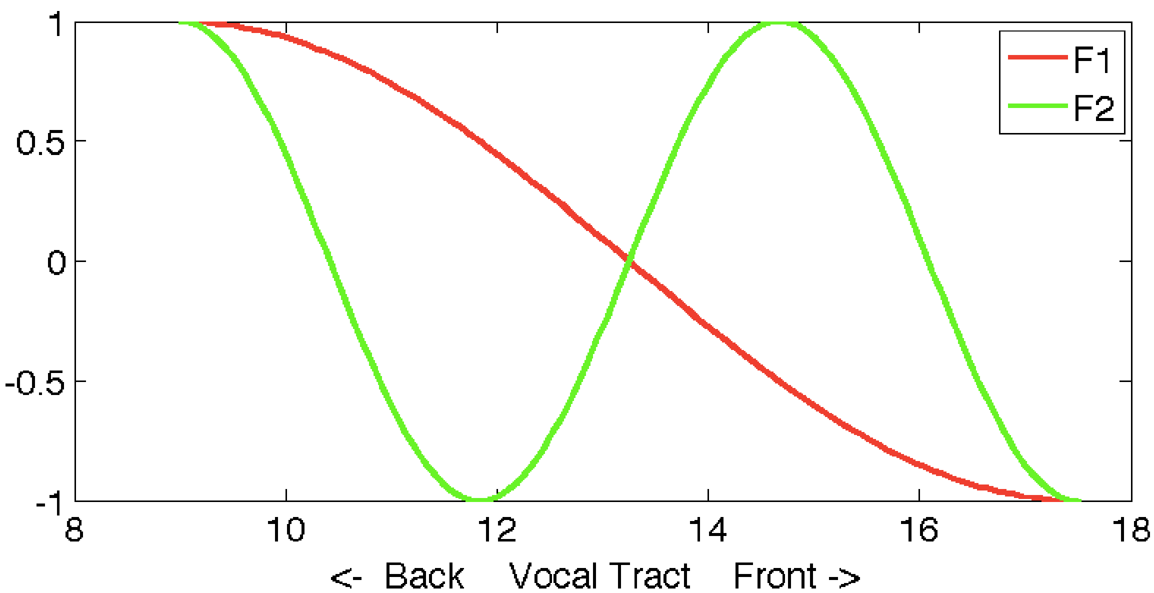
$$f_n = \frac{nc}{4L}$$

$$.175m \longrightarrow 500\ Hz, 1500\ Hz, 2500\ Hz$$

B) Perturbation

- The perturbation itself can be expressed as a list of numbers (vector), with 0's through it all, except for the point in space at which a perturbation exists, e.g.:
- Simple /a/: [0 0 0 **1** 0 0 0 0 0 0 0 0 0]
- Simple /i/: [0 0 0 0 0 0 0 0 0 **1** 0 0 0]
- Simple /u/: [0 0 0 0 0 0 **1** 0 0 0 0 0 **1**]

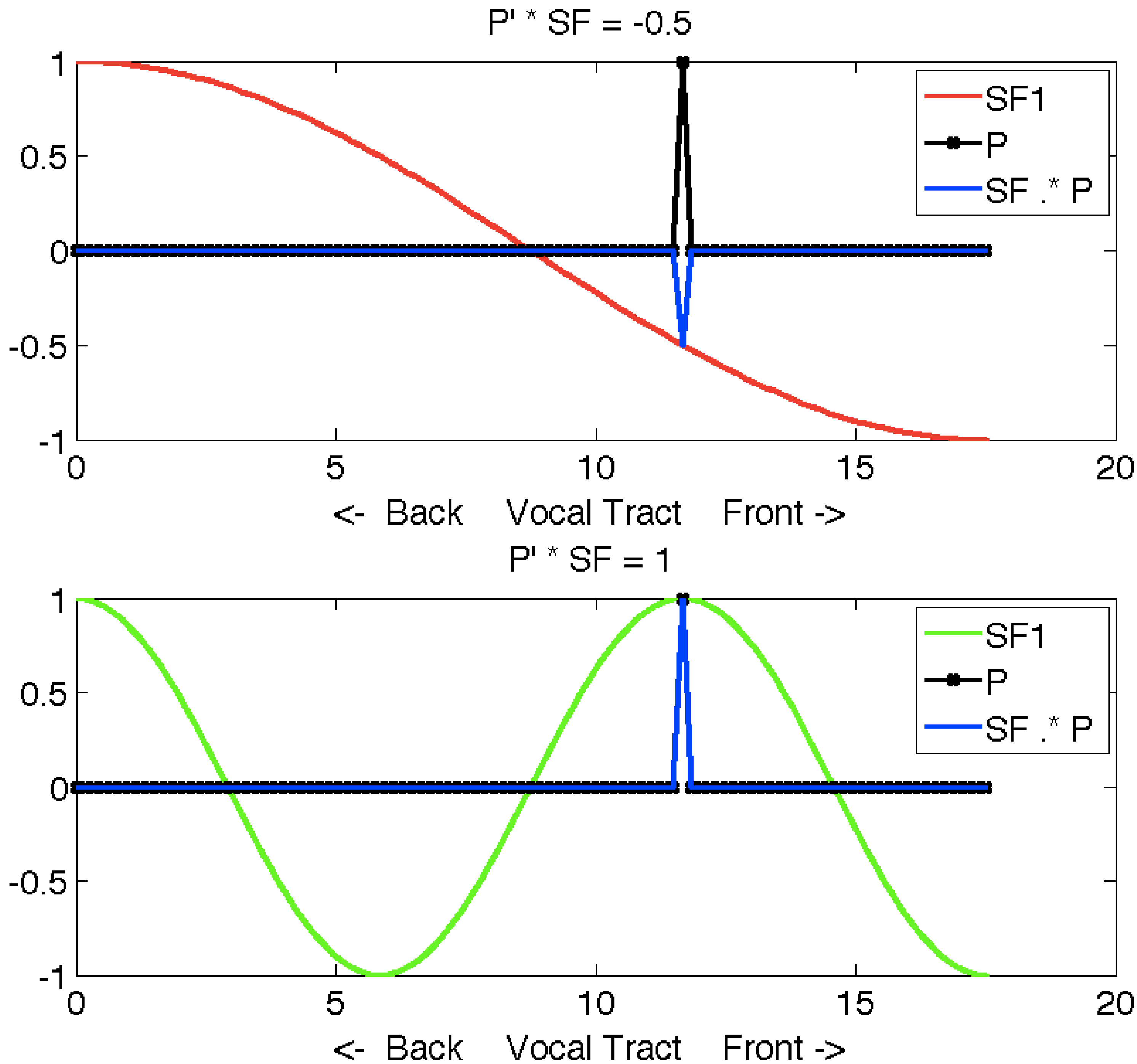
- C) <u>Sensitivity Function</u>: a mathematical statement of the physical logic we went through:

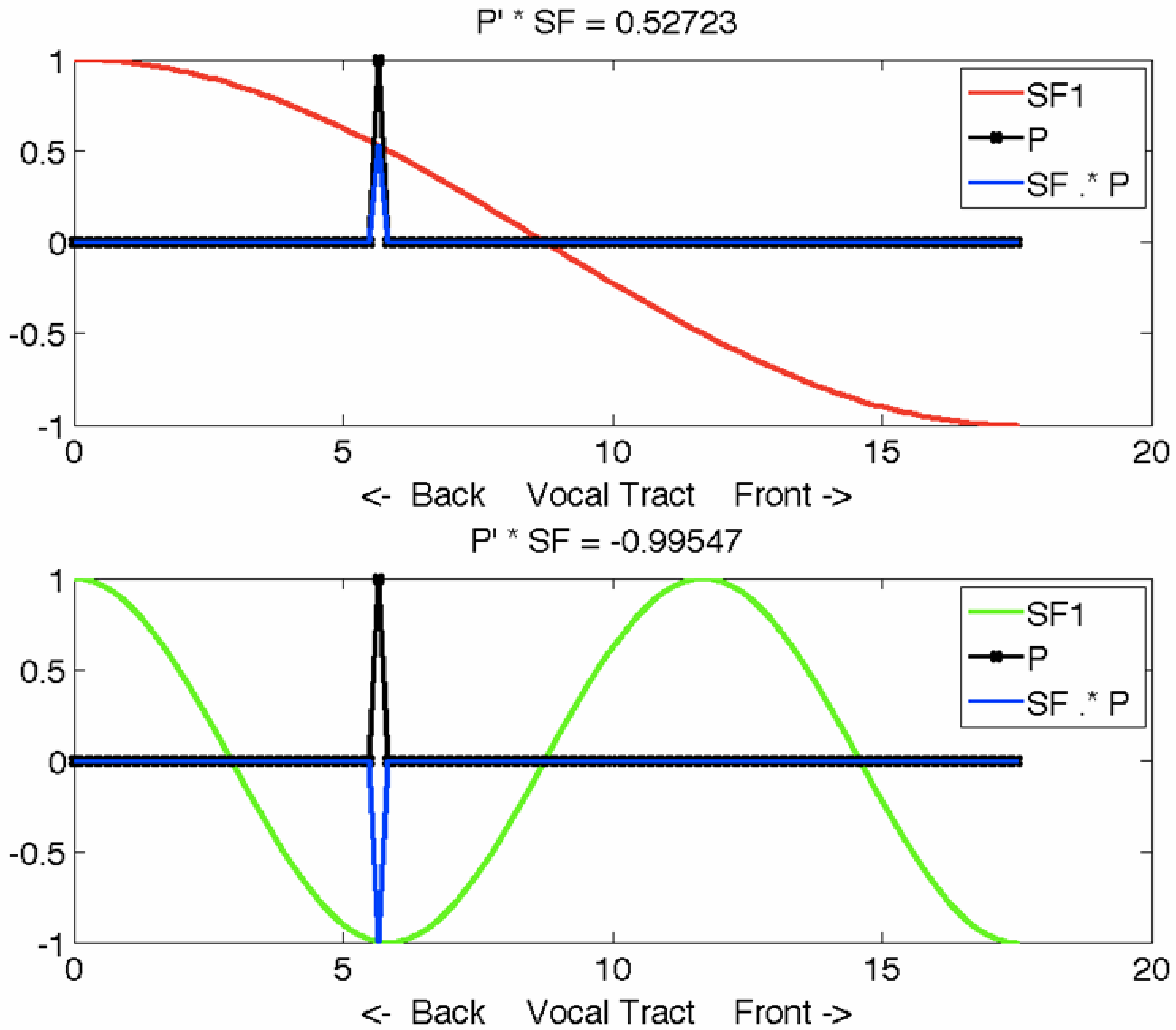


	m$\uparrow$k$\uparrow$
1) Open End	All F_i decrease
2) Closed End	All F_i increase
3) 1/3	$F_{1:}$: Mid $\uparrow$ F_2: Max $\downarrow$
4) 2/3	F_1: Large $\downarrow$ F_2: Max $\uparrow$
5) Mid	F_1: No Eff. F_2: No Eff.

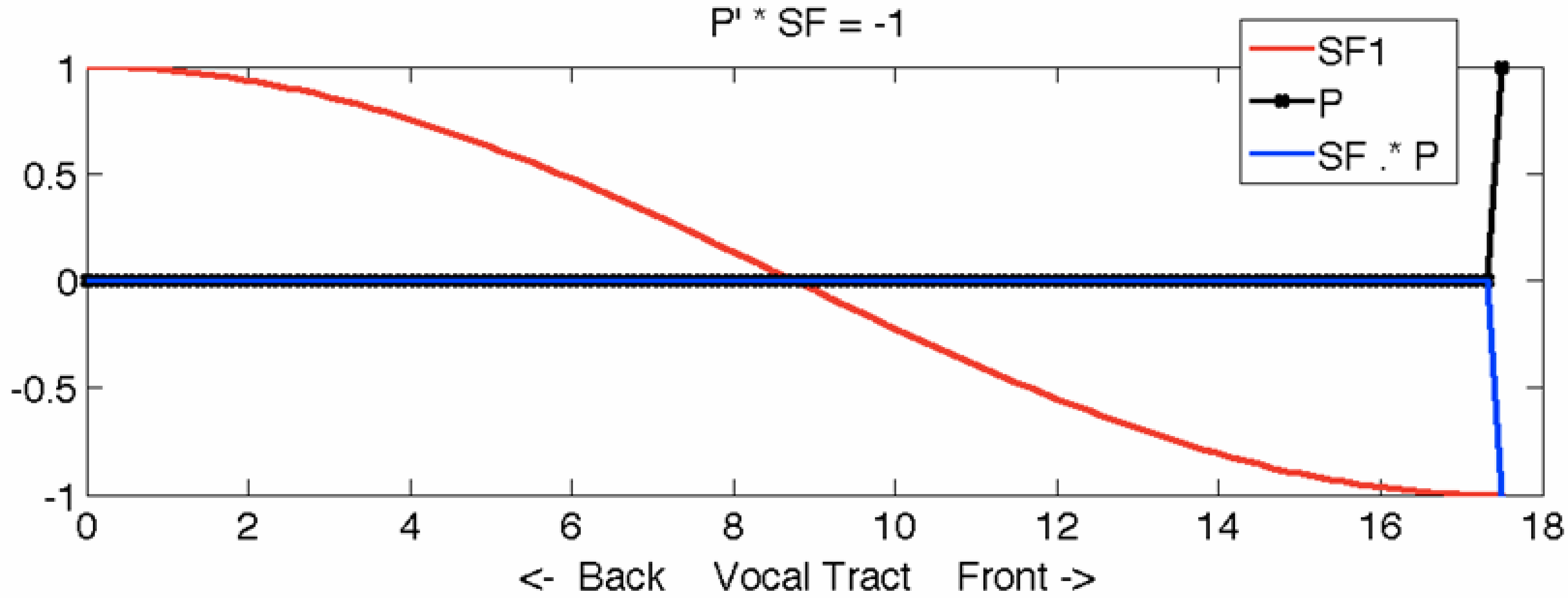
Weigh Perturbation Function by SF

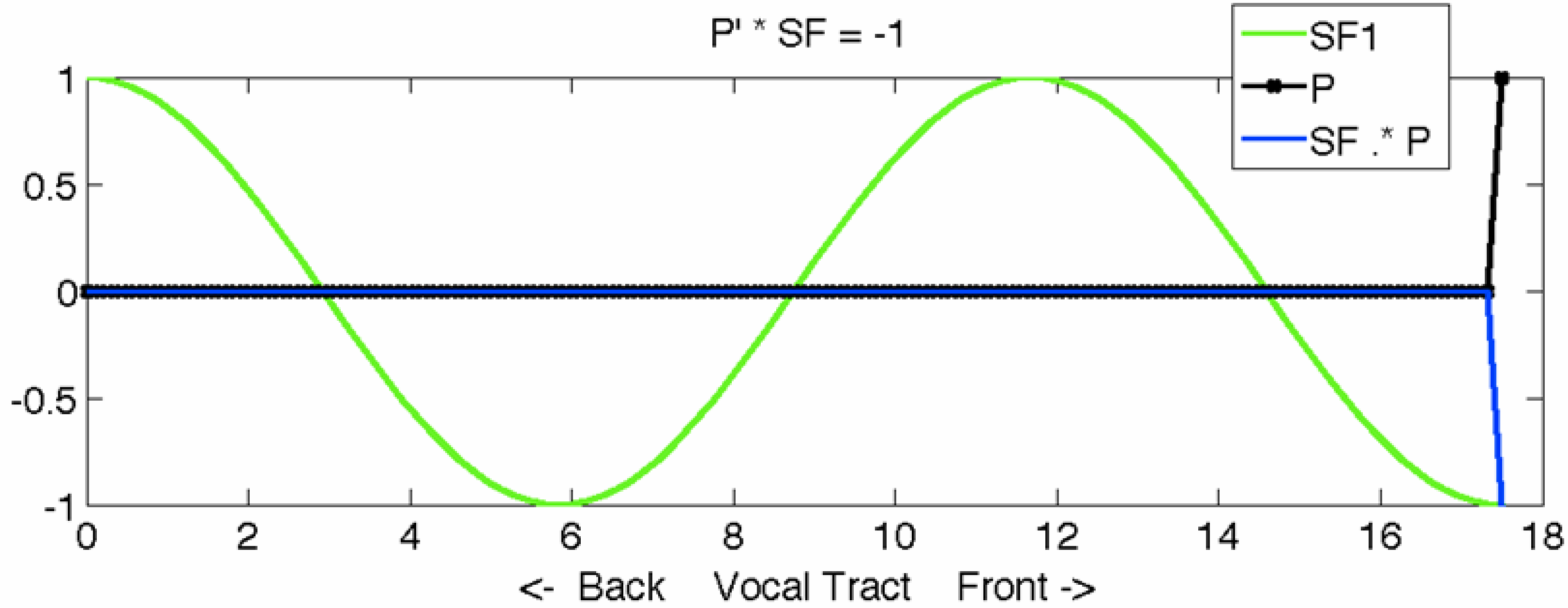
- Now we have two functions of x, the perturbation function (P) and the sensitivity function (SF).
- At each x, multiply the one by the other: P .* SF and sum over x (inner product):
  - P' * SF
- The points at which PF = 0, don't contribute to the inner product.
- x-points at which PF = 1 for which SF = 1, will contribute a 1 to the sum and should increase freq.
- x-points at which PF = 1 for which SF = -1, will contribute a -1 to the sum and should decrease freq.


P' * SF = -0.5
SF1
P
SF .* P
1
0.5
0
-0.5
-1
0
5
10
15
20
<- Back Vocal Tract Front ->
P' * SF = 1
SF1
P
SF .* P
1
0.5
0
-0.5
-1
0
5
10
15
20
<- Back Vocal Tract Front ->



P' * SF = 0.52723
SF1
P
SF .* P
<- Back Vocal Tract Front ->
P' * SF = -0.99547
SF1
P
SF .* P
<- Back Vocal Tract Front ->


P' * SF = -1
SF1
P
SF .* P
1
0.5
0
-0.5
-1
0
2
4
6
8
10
12
14
16
18
<- Back Vocal Tract Front ->


P' * SF = -1
SF1
P
SF .* P
1
0.5
0
-0.5
-1
0
2
4
6
8
10
12
14
16
18
<- Back Vocal Tract Front ->

The Formula (Ehrenfest's Theorem)

- New Formant Freq. =

  Neutral Formant Freq. + (

  (P' * SF)/2 * Neutral Formant Freq.)

$$F_i = NF_i + \frac{(P'*SF)}{2} NF_i$$

	Neutral Freq	P’ * SF	New Freq
/i/	500 1500	-.5 1	375 2250
/a/	500 1500	.5 -1	625 750
/u/	500 1500	-1 -1	375 750
schwa	500 1500	0 0	500 1500