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A THEORETICAL ANALYSIS OF AIRFLOW IN A CONVERGENT GLOTTIS

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ABSTRACT

By application of a common theory of fluid mechanics to the convergent glottis the following equations for aerodymanic quantities were established: velocity and pressure distributions, pressure drop across the glottis. The effects of the viscosity of the air and of the area change could be described analytically due to the theoretical approach, which is based on the Navier-Stokes equations and the continuity equation. The derivation of analogous equations for the divergent glottis was not possible due to the general instability of the flow in this glottal configuration.

1 INTRODUCTION

The phonatory process depends on tissue properties of the vocal folds, interactions between glottal airflow and the vocal folds, and the acoustic coupling of the glottis to the sub- and supraglottal tract cavities. In order to sustain vocal fold oscillations the natural damping of energy by friction in the tissue has to be overcome, which is achieved by a transfer of energy form the airflow to the vocal folds. A necessary condition for this energy transfer is an asymmetry of the driving pressure with respect to the opening and the closing phase of a glottal cycle. This asymmetry is mainly achieved by a periodic change of convergent and divergent glottal configurations [Titze, 1993].

In this paper the mechanics of laminar flow in a convergent glottis will be analyzed, which may be regarded as one step towards the investigation of the above mentioned interactions occuring in voice production. The divergent glottis will only be considered for low Reynolds numbers as otherwise the laminar flow would be unstable.

2 GEOMETRICAL REPRESENTATION OF THE GLOTTAL CONFIGURATIONS

For the analysis of glottal flow mechanics a well-known theory of laminar flow between nonparallel plane walls [Jeffrey, 1915][Hamel, 1916][Landau and Lifshitz, 1991], which is based on the stationary Navier-Stokes equations and the continuity equation, is applied to the glottis. Thereby the glottis is the space between two geometrical planes, which represent the rims of the vocal cords (see fig. 1). Two basically different cases of glottal geometries will be considered:

1. The planes intersect (the vocal cords are in contact at the beginning or at the end of the glottis). For the divergent glottis there is a source (of flow) and for the convergent glottis a sink at the intersection line of the planes. The origin of the coordinate system lies at one end of the intersection line. This is the geometry used in the above mentioned theory.
2. During phonation the vocal cords are not always in contact but have a minimal distance from each other, which means that the planes do not intersect. For the description of this case the origin of the coordinate system lies at one end of the virtual intersection line of the planes and two values r_1 and r_2 are defined that represent the minimal radial distances of the origin form the starting and ending points of the glottis along the vocal cords (see fig. 1). This change of the geometry (with respect to the first case) implies different boundary conditions of pressure and velocity. The theory is only applicable for small minimal distances of the vocal cords and large glottal angles as otherwise the virtual intersection line would be too far away from the glottis.

The glottal flow will be analyzed in the two-dimensional r-φ-plane (polar coordinates are used to simplify the description) whereas it will be assumed homogeneous in the z-direction (direction of the glottal length). From the mathematical point of view the two configurations (convergent and divergent) differ only with respect to the signs of the radial distances and the velocities.

Due to the symmetry of the flow region with respect to the bisector of the angle the laminar flow can be assumed to be purely radial:

$$v_r = v_r(r,\varphi), \quad v_\varphi = v_z = 0$$

Because of the independent investigation of these different

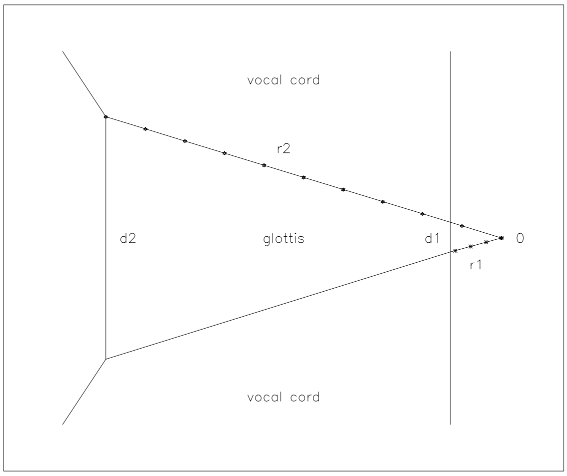

Abbildung 1: Geometrical representation of the (convergent) glottis as a space between two nonparallel planes (rims of the vocal cords) with a glottal angle α. 0: origin of the coordinate system; d_1, d_2: diameters at the beginning and the end of the glottis; r_1 und r_2: radial distances from the origin to the beginning and the end of the glottis

glottal forms the quasistatic approximation for the glottal flow is required. The validity of this approximation has been investigated theoretically [Flanagan, 1958] and experimentally [Mongeau et al., 1992]; the approximation is, however, not applicable to all conditions of phonation.

3 VELOCITY DISTRIBUTIONS

Using the above mentioned theory, the velocity distribution of laminar flow in the convergent or divergent glottis can be described in a general form by the following equations (for small Reynolds numbers) [Landau und Lifschitz, 1991] [Hamel, 1917]:

$$v_r(r,\varphi) = \frac{u(\varphi)6\nu}{r}, \tag{1}$$

$$u(\varphi) = u_o + \wp(\varphi - \varphi_o, g_2, g_3) \tag{2}$$

$\wp$: elliptic function of Weierstrass (for a definition see e.g. [Abramowitz and Stegun, 1972])
u_o, φ_o, g_2, g_3: constants
$\nu(= \frac{\eta}{\rho})$: kinematic viscosity

The constants can be determined from the boundary conditions.

These profiles change for higher Reynolds numbers (e.g. those relevant for phonation). It has been proved that the divergent flow becomes unstable whereas the convergent flow remains stable even for values of the glottal angle up to 180^o and unlimited Reynolds numbers [Hamel, 1916]. The divergent flow separates from the wall, whereby the point of separation can be calculated [Pohlhausen, 1921].

As the divergent flow is not stable for all Reynolds numbers only the convergent flow will be considered in the following calculations.

4 PRESSURE DISTRIBUTION

The pressure distribution in a convergent glottis is given by the following equation [Landau and Lifshitz, 1991]:

$$p(r,\varphi) = \frac{6\eta^2}{\rho}\frac{(2u(\varphi) - C_1)}{r^2} + C_2 \tag{3}$$

ρ: density
η: viscosity

The radial pressure distribution at the vocal cords can be derived from this equation as $u\left(\pm\frac{\alpha}{2}\right) = 0$:

$$p(r) = p_o - \frac{6\nu^2}{\rho}\frac{C_1}{r^2} \tag{4}$$

p_o: subglottal pressure

In order to consider the influence of the supraglottal space the variable r may be replaced by $r - r_o$ so that the curve (or rather of the pole of the curve) $p(r)$ is shifted with respect to the theoretical origin (see fig. 1) with the help of the empirically determinable distance r_o. The upper limit of validity of eq. refeq:pdisrtph) and (4) is the coordinate r_{nd} where the value of the pressure becomes zero (defined value of the supraglottal pressure) because beyond this point pressure fluctuations may take place until the pressure has stabilized; these possible fluctuations are not described here. The total pressure acting upon the vocal folds can be calculated by integrating eq. (4).

5 PRESSURE DROP ACROSS THE GLOTTIS

The pressure drop across the convergent glottis can be described by the following equation (see appendix for the derivation):

$$\begin{aligned} \Delta P &= \Delta P_r + \Delta P_\varphi \\ &= \frac{6\eta^2 C_1}{\rho}\left(\frac{1}{{r_1}^2} - \frac{1}{{r_2}^2}\right) \\ &\quad -2\eta\left(\frac{1}{r_1 A_1} - \frac{1}{r_2 A_2}\right) U + C_2 \end{aligned} \tag{5}$$

ΔP_r: pressure drop due to the purely radial distribution of the pressure

ΔP_φ: pressure drop due to the angular distribution of the pressure
r_1, r_2: s. fig. 2
A_1, A_2: glottal areas at the beginning and at the end of the glottis
C_1, C_2: constants

This equation can be interpreted as a generalization of the well-known formula for the same relationship in a rectangular glottis:

$$\Delta P = \frac{12\eta L^2 T}{A^3} = \frac{12\eta T}{d^2 A}\, U \tag{6}$$

T: glottal thickness
d: glottal diameter
L: glottal length
A: glottal cross-sectional area

The pressure drop is described by two terms because it not only depends on the volume velocity but also on the change of the cross-sectional area. The two terms in eq. (5) and (6) depending on the volume velocity are similar.

The relationship between the pressure drop and the volume velicity can be obtained by inserting eq. (5) into the Bernoulli equation:

$$\begin{aligned} \Delta P &= \frac{1}{2}\rho\left(\frac{1}{{A_1}^2} - \frac{1}{{A_2}^2}\right)U^2 \\ &= \frac{6\eta^2 C_1}{\rho}\left(\frac{1}{{r_1}^2} - \frac{1}{{r_2}^2}\right) \\ &\quad -2\eta\left(\frac{1}{r_1 A_1} - \frac{1}{r_2 A_2}\right)U + C_2 \end{aligned} \tag{7}$$

$\Longrightarrow$

$$\begin{aligned} \Delta P_r &= 2\eta\left(\frac{1}{r_1 A_1} - \frac{1}{r_2 A_2}\right)U \\ &\quad +\frac{1}{2}\rho\left(\frac{1}{{A_1}^2} - \frac{1}{{A_2}^2}\right)U^2 \\ &= \frac{6\eta^2 C_1}{\rho}\left(\frac{1}{{r_1}^2} - \frac{1}{{r_2}^2}\right) + C_2 \end{aligned} \tag{8}$$

6 DISCUSSION

Apart from several equations describing pressure-flow relationships in the glottal region with a rectangular glottis ([Wegel, 1930][van den Berg et al., 1957][Ishizaka and Flanagan, 1972][Ishizaka and Matsudaira, 1972]) there have been developed some equations where the same relations were described for convergent and divergent glottal geometries. Ishizaka and Matsudaira [1972] approximated the convergent and the divergent glottis with two rectangluar ducts of different diameters and considered the pressure drop due to the sudden cross-sectional change with the Bernoulli equation:

$$\begin{aligned} \Delta P &= \left(\frac{12\eta L^2 T_1}{{A_1}^3} + \frac{12\eta L^2 T_2}{{A_2}^3}\right)U \\ &\quad +\frac{1}{2}\rho\left(\frac{1}{{A_2}^2} - \frac{1}{{A_1}^2}\right)U^2 \end{aligned} \tag{9}$$

This equation is only valid for small angles but it has been complemented to be also adjustable to experimental data for big angles [Ishizaka, 1985]. Thereby the Bernoulli equation was generalized in such a way that changes of the pressure drop due to area changes and the viscosity could be considered using empirical factors (here the equation for the convergent glottis is given):

$$\begin{aligned} p_1 + \frac{1}{2}\rho {v_1}^2 &= p_2 + \frac{1}{2}\rho {v_2}^2 \\ &\quad +\eta_c\left(1 - \frac{A_2}{A_1}\right)^2 \frac{1}{2}\rho {v_2}^2 + h_v \end{aligned} \tag{10}$$

h_v: pressure drop due to viscosity
η_c: empirically determinable factor

With these equations experimental and numerical data of pressure-flow relationships can be quite well described [Scherer und Titze, 1983][Gou und Scherer, 1993]. These equations are complemented by eq. (8) as the pressure drop is entirely derived from the Navier-Stokes equations and the continuity equation and therefore the changes of the pressure drop due to the glottal geometry and the viscosity are expressed analytically considering the exact geometry (A_1 and A_2 are exchanged due to the different relative position of the origin of the coordinate system). However it is only applicable to the convergent glottis whereas eq. (9) and (10) can also be applied to the divergent glottis. The main advantage of these new equations (5) and (8) is that additionally to the pressure-flow relationship the distributions of velocity and pressure can also be computed. Thereby data of aerodynamic quantities obtained with different experimental or theoretical methods (e.g. experiments with static glottal models or numerical simulations) can be analyzed with one analytical approach. With these data the validity of the new equations can be examined.

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A CALCULATION OF THE PRESSURE DROP ACROSS A CONVERGENT GLOTTIS

First the volume velocity is calculated ($\vec{v}$ is oriented from r_2 to r_1 (see fig. 1)) which is then inserted into the equation describing the pressure drop. The volume velocity is obtained by integrating equation (1) (radial component of the velocity) over φ and z and averaging over r (this means integration over r and division by $\int dr = r_1 - r_2$).

$$\begin{aligned} U &= \frac{1}{r_1 - r_2} \int_0^L \int_{-\frac{\alpha}{2}}^{+\frac{\alpha}{2}} \int_{r_2}^{r_1} v(r,\varphi) r \, dr \, d\varphi \, dz \\ &= \frac{L}{r_1 - r_2} \int_{-\frac{\alpha}{2}}^{+\frac{\alpha}{2}} \int_{r_2}^{r_1} 6\nu u(\varphi) \, dr \, d\varphi \\ &= 6L\nu \int_{-\frac{\alpha}{2}}^{+\frac{\alpha}{2}} u(\varphi) \, d\varphi \end{aligned}$$

$\Longrightarrow$

$$\int_{-\frac{\alpha}{2}}^{+\frac{\alpha}{2}} u(\varphi) \, d\varphi = \frac{U}{6L\nu} \tag{11}$$

To calculate the pressure drop the function $p(r)$ is determined by averaging $p(r,\varphi)$ over the area $(r \, d\varphi) \, dz$ with the variable r kept constant and inserting equation (11):

$$\begin{aligned} p(r) &= \frac{\int_0^L \int_{-\frac{\alpha}{2}}^{+\frac{\alpha}{2}} \left(\frac{6\eta^2}{\rho r^2}(2u(\varphi) - C_1) + C_2 \right) r \, d\varphi \, dz}{\int_0^L \int_{-\frac{\alpha}{2}}^{+\frac{\alpha}{2}} r \, d\varphi \, dz} \\ &= \frac{1}{r\alpha} \left(\frac{12\eta^2}{\rho r} \int_{-\frac{\alpha}{2}}^{+\frac{\alpha}{2}} u(\varphi) \, d\varphi - \frac{6\eta^2 C_1 \alpha}{\rho r} + C_2 r\alpha \right) \\ &= \frac{12\rho\nu^2}{r^2\alpha} \frac{U}{L6\nu} - \frac{6\eta^2 C_1}{\rho r^2} + C_2 \\ &= \frac{2\eta}{rA} U - \frac{6\eta^2 C_1}{\rho} \frac{1}{r^2} + C_2 \end{aligned}$$

$A = (r\alpha) \, L$

The pressure drop ΔP is obtained by determining the pressure difference of $p(r)$ at r_2 and r_1 :

$$\begin{aligned} \Delta P &= p(r_2) - p(r_1) \\ &= \frac{6\eta^2 C_1}{\rho} \left(\frac{1}{{r_1}^2} - \frac{1}{{r_2}^2} \right) \\ &\quad -2\eta \left(\frac{1}{r_1 A_1} - \frac{1}{r_2 A_2} \right) U + C_2 \end{aligned}$$