Introduction


Figure: (top) Figures 1 (left) and 3 (center) from Browman and Goldstein lagharghi ( 1993 ) (left) and Figure 3 (right) from Irino and Patterson (2002).

- Vowel dynamics is a complex concept that branches across three broad areas of speech and language research:
Articulatory Dynamics: the specification of the motion of articulators during the production of vowels, typically described at several levels of motor control,
Acoustical Dynamics : the formulation of acoustic measures that are meant to encode the transitional aspects of familiar "static" acoustic properties,
Auditory Dynamics: the modeling of the perceptual capacities of the auditory system, which capture its sensitivity to a number of temporal aspects of sound internalization.
While much insight has been achieved, very little is known about how infants acquire cognitive structures that undergird the cognitive representation of vowel dynamics.


## Cognitive Manifolds and Graph Methods


-We put forward a computational model of the emergence of cognitive structures, called cognitive manifolds, that facilitate the acquisition of vowel dynamics during early infancy and are derived from an infant's vocal interaction with caregivers.

- The principle computation within the framework, called manifold alignment (Lee et al. 2005, Wang, 2010), generates new manifolds that yield mappings of vowel production representations onto structures that facilitate, inter alia an infant's representation of vowel-vowel segmental combinations, and
an infant's formation of preliminary relations between their own vowel-vowel sequences and those of their caregivers. Moreover, the structural operations provided by the formation of aligned manifolds based on vowel production representations the creation of units that may enter into paradigmatic structures.

Modeling the emergence of cognitive structures for the acquisition of vowel dynamics during early infancy using manifold alignment Andrew R. Plummer

Manifold Formation and Laplacian Eigenmapping

biectively mapped to vertex sets $V_{1}$ and bijectively mapped to vertex sets $V_{1}$ and $V_{2}$
denoted, $m_{1}: X_{1} \rightarrow V_{1}$ and $m_{2}: X_{2} \rightarrow V_{2}$
Manifolds derived from $X_{1}$ and $X_{2}$ are weighted graphs $G_{1}=\left(V_{1}, E_{1}\right)$ and weighted graphs $G_{1}=\left(V_{1}, E_{1}\right)$ and
$G_{2}=\left(V_{2}, E_{2}\right)$ formed over $V_{1}$ and $V_{2}$ using,
es e.g., a nearest neighbors computation over
$V_{1}$ and
3. The manifolds $G_{1}$ and $G_{2}$ are combined into a new manifold $G_{1 \otimes 2}$ via a set $X \subset V_{1} \times V_{2}$
of alignment edges (purple) weighted according to the importance of the pairing.
. Data sets $X_{1}$ and $X_{2}$ are eigenmapped (Belkin and Niyogi, 2003) to a new reference frame which reflects local relations captured
by $E_{1}$ and $E_{2}$, and the alignment $\chi$ encodes.
$-0.50-0.05$
$0.50-$
${ }_{0.57}^{0.50}$.
(8) ${ }^{1010}$


Paths and Trajectories


A path graph of order $n$, denoted $P_{n}$, is a graph with $n$ vertices, exactly two of A path graph of order $n$, denoted $P_{n}$, is a graph with $n$ vertices, exactly two of
which have degree 1 , while all others have degree 2. A path of order $n$ in a graph $G=(V, E)$ is a subgraph of $G$ that is graph isomorphic to $P_{n}$.
While paths are not inherently directional, directionality can be attributed to a path $p$ by treating it as a sequence $v_{1} v_{2} \cdots v_{n}$, called a path from $v_{1}$ to $v_{n}$,
denoted $p_{v, ~}, v_{1}$ where $v_{1}$ is called initial and $v_{n}$ is terminal.

Let $M_{x}=\left(V_{x}, E_{x}\right)$ be a manifold derived from a data set $X$. Given a path $p=v_{1} v_{2} \cdots v_{n}$ in $M_{x}$, a trajectory $t_{p}$ corresponding to $p$ is a function of the sequence $t=l_{1} t_{2} \cdots t_{n}$ where $t_{i}=m\left(v_{i}\right)$.
Paths $p_{1}=v_{1} v_{2} \cdots v_{n}$ and $p_{2}=w_{1} w_{2} \cdots w_{k}$ where $w_{1}=v_{n}$ can be concatenated to form a path cat $\left(p_{1}, p_{2}\right)=v_{1} v_{2} \cdots v_{n} w_{2} \cdots w_{k}$ from $v_{1}$ to $w_{k}$.
Concatenated paths naturally have corresponding concatenated trajectories. Given a path $p=v_{1} v_{2} \cdots v_{n}$, the reverse of $p$ is the path $\operatorname{rev}(p)=v_{n} v_{n-1} \cdots v_{2} v_{1}$ from $v_{n}$ to $v_{1}$. Reversed paths naturally have corresponding reversed trajectories.
A shortest path from $v$ to $w$ is a path $p=v \cdots w$ such that the order of $p$ is less than or equal to the order of all other paths from $v$ to $w$. The existence
of a path $p=v \cdots w$ guarantees the existence of a shortest path from $v$ to $w$.

Paths and Trajectories in Generated Frames Data and Alignment Pairs

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Manifolds are formed over data sets $Q$ (magenta) and $Q_{2}$ (blue) in the emanner
descosired at teft, and they are aligned using
d set ot pais

2. Maniiolds $M_{Q_{2}}$ and $M_{Q_{2}}$ can then be formed
over $\alpha_{1}$ an $\alpha_{2}$, and used to construct paths nd corresponding trajectories connecting regions corresponding to $\chi$ oor.


From Syntagmatic Paths to Paradigmatic Units


Let $M_{x}=\left(V_{x}, E_{x}\right)$ be a manifold derived from a data set $X$. A target set $T \subseteq V_{X}$ is a collection of target vertices that have special significicance, e.g.
by corresponding to data in $X$ coming from turn-taking vocal exchange.
Let $u$ and $i$ be target vertices in $T$. A path $p$ from $u$ to $i$ is called a syntagmatic path in $M_{x}$, and a trajectory $t_{p}$ is a syntagmatic trajectory The target subsequence of a path $p$ from $u$ to $v$ in $M_{x}$, denoted $\operatorname{tar}(p)$, is a subsequence of $p$ composed of the target vertices in $p$.
Let's say that two target vertices $u$ and $i$ are separable iff there exist paths $p_{1}$ and $p_{2}$ such that $u$ and $i$ are the initial and terminal vertices of $p_{1}$ and $p_{2}$, espectively, and $\operatorname{tar}\left(p_{1}\right) \neq \operatorname{tar}\left(p_{2}\right)$.
Let's consider four target vertices $u, i, o$, and $e$, and suppose we have paths
$p_{u e}$ (traj). in orange), $p_{i \text { ( }}$ (blue), and $p_{i e}$ (red). We can form the path

$$
p_{\text {ueio }}=\operatorname{cat}\left(\operatorname{cat}\left(p_{u e}, r e v\left(p_{i e}\right)\right), p_{i o}\right)
$$

fom $u$ to o (green, bottom left)
The path $p_{u e i}$ guarantees the existence of a shortest path $p_{w o}$, and the nature of $M_{x}$ ensures that $u$ and $o$ are separable, and each vertex in $T$ is separable from each of the others.
A set of target vertices $T$ is a paradigmatic set if each vertex $v \in T$ is is called a paradigmatic unit.
(Auditory-based) Commensuration Structure


Given a set of socio-vocal data from turn-taking exchanges (top), together with a broad set of internalized vocal experience and interna with a broad set of internalized vocal experience and internal aniflds which generate representations for the creation of structures for the acquisition of vowel dynamics.

To illustrate, we'll simplify the picture a bit, and focus on the internaizzation corner vowels [i], [a], and [u], and the formation of auditory manifolds form over auditiory rep
The infant aligns these manifolds using the internalized turn-taking experience, yielding (auditory-based) commensuration representations that are used to form a commensuration manifold.

The turn-taking experience also yields the target regions for syntagmatic ath construction through the commensuration manitold which in turn the paradigmatic units that may enter into a phonology.

## Discussion

Much attention has been paid to computational models of phonological acquisition in recent decades, with renewed vigor in the subject seemingly occurring in concert with the promulgation of the latest algorithm/computation that catches the attention of funding agencies and other kingmakers.
There is no reason to limit our view to what can be easily quantified and computed using popular computational methods when it is quite valuable to attempt to use an array of methods bring within our view areas of inquiry that are scarcely ecognized.
At present, we are attempting to use a recent variation on the graphical model approach to shed light on the nature of the acquisition of cognitive structures for the representation of vowel dynamics, and its potential relations to an infant's development of syntagmatic and paradigmatic systems of speech and language analysis.

## Acknowledgments

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