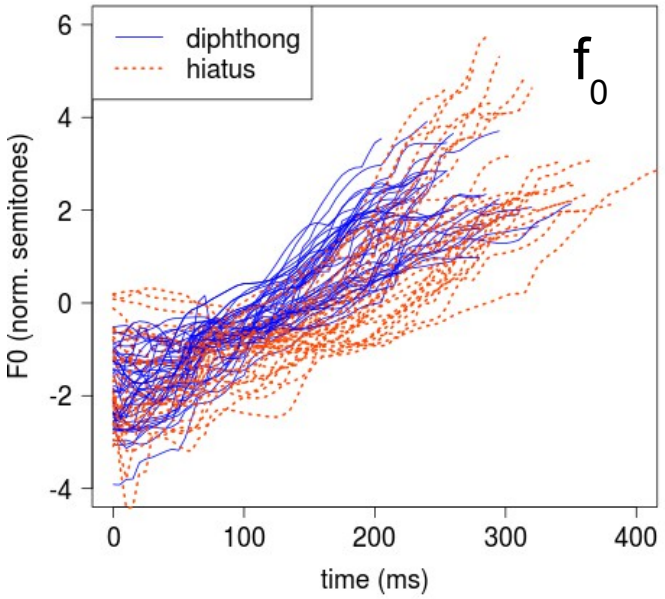
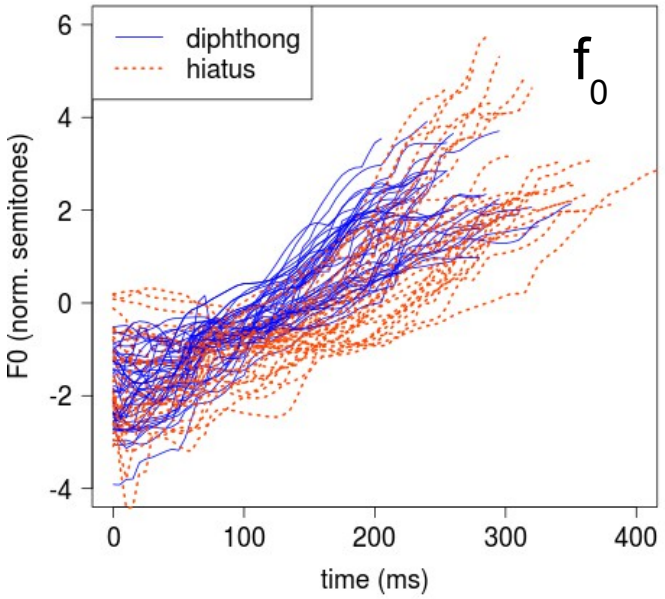
Functional Data Analysis (FDA) for phonetic research

Michele Gubian
University of Bristol, UK



- European Spanish
- **Diphthong**: /ja/
- **Hiatus** /i.a/
- Rising pitch accent
- Tonal alignment?



ANOVA

LR

LMER

- European Spanish
- **Diphthong**: /ja/
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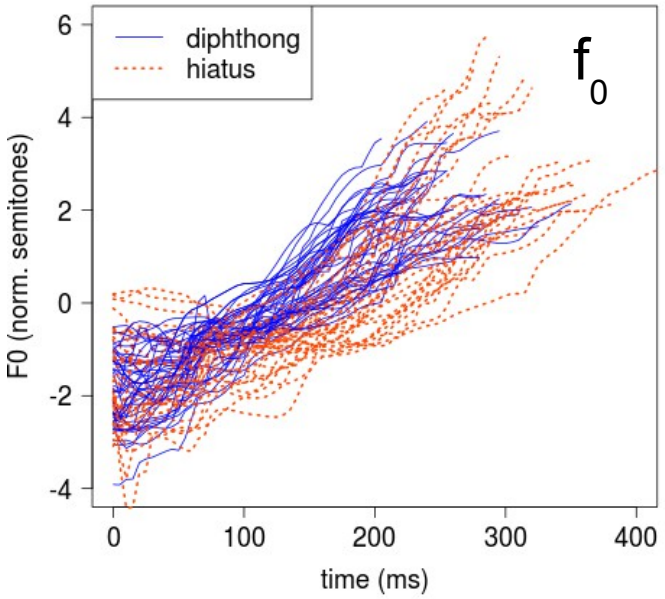
CURVES



NUMBERS

- ANOVA
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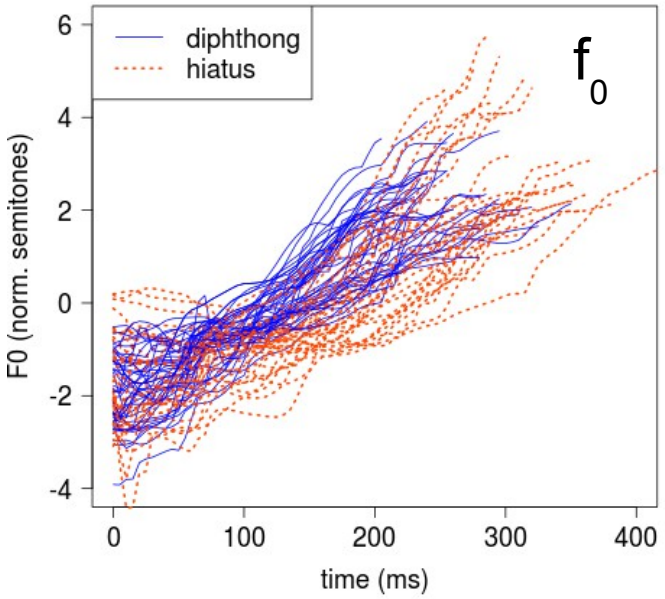

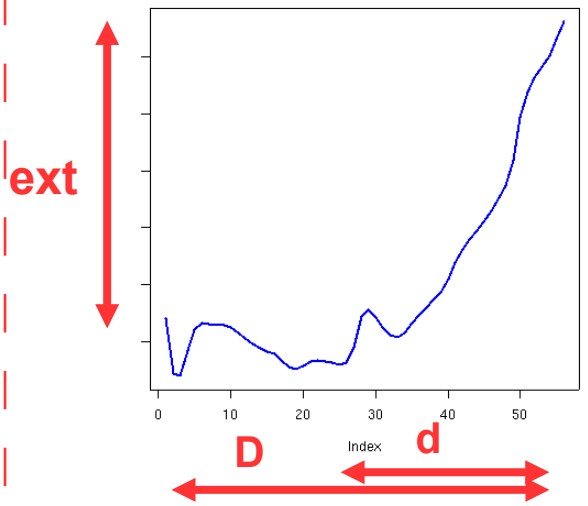

ext (st)	d/D	Cat.
5.3	0.9	D
4.6	0.7	H
...	...	...

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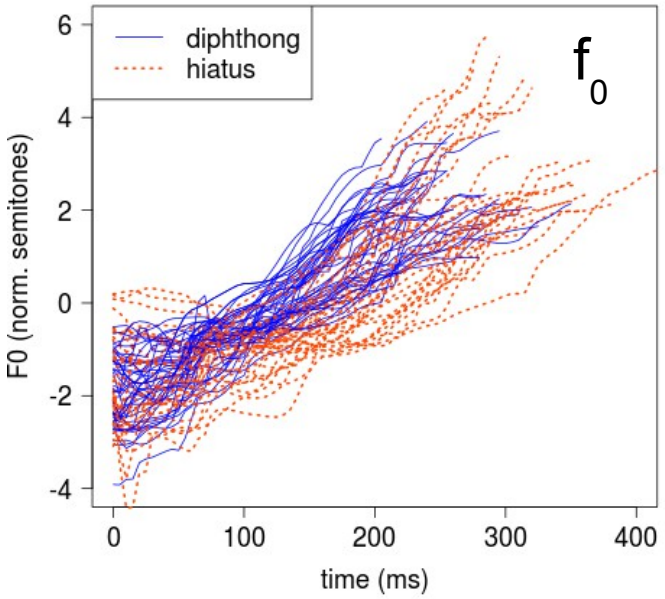

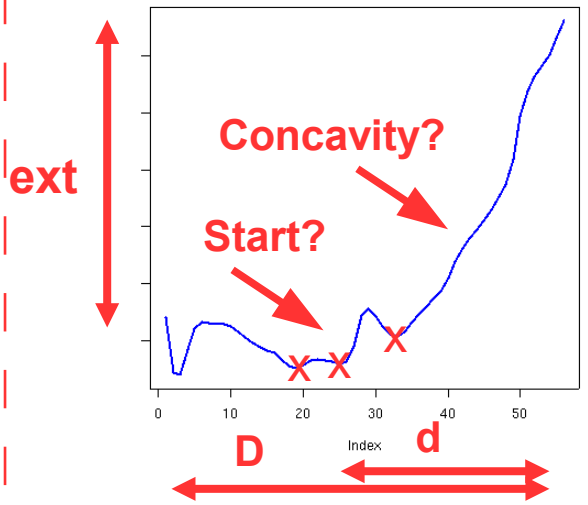

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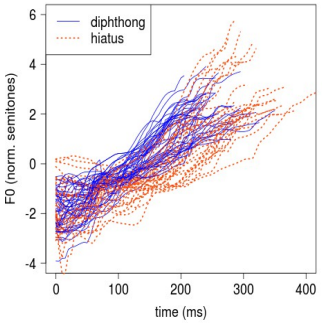
MISSION

automate curve parametrisation

- Data driven
- Few parameters
- Interpretable

Road map

CURVES



Interpolate using a function basis

- Data driven

Dimensionality reduction tool

- Few parameters
- Interpretable

NUMBERS

- ANOVA
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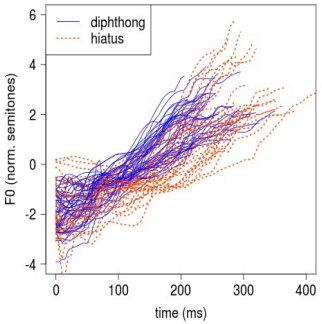
Road map

CURVES



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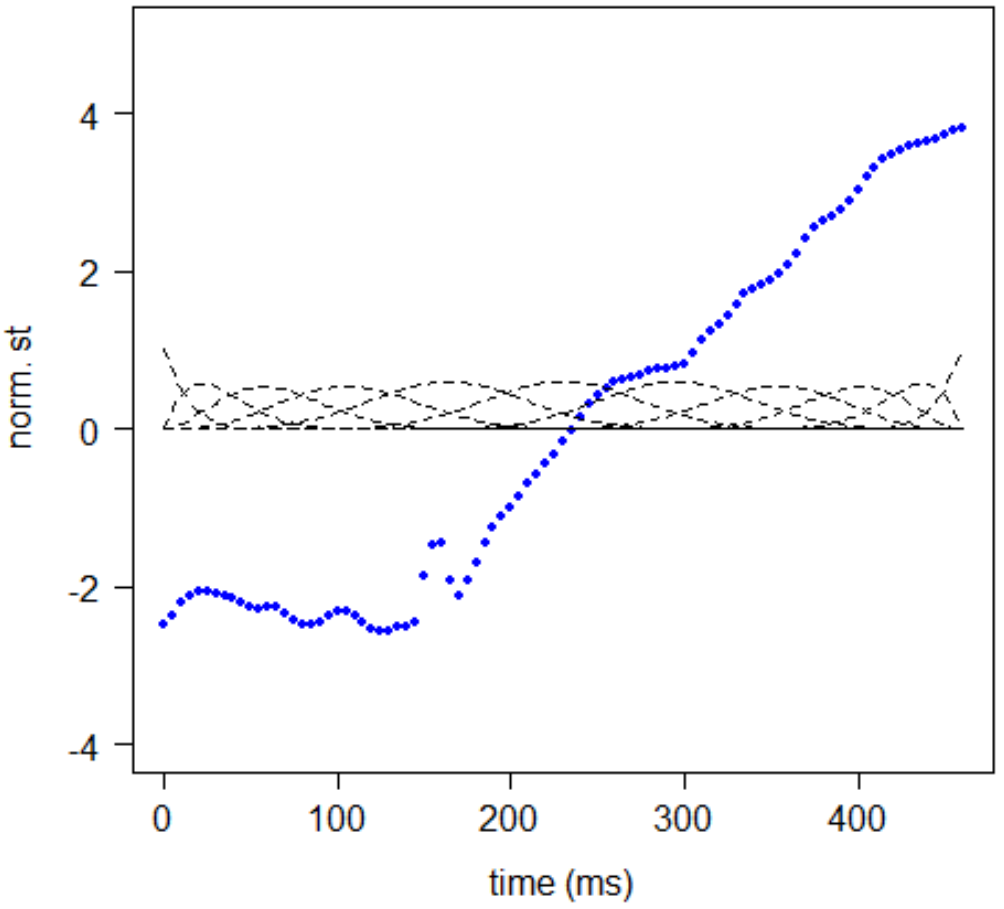
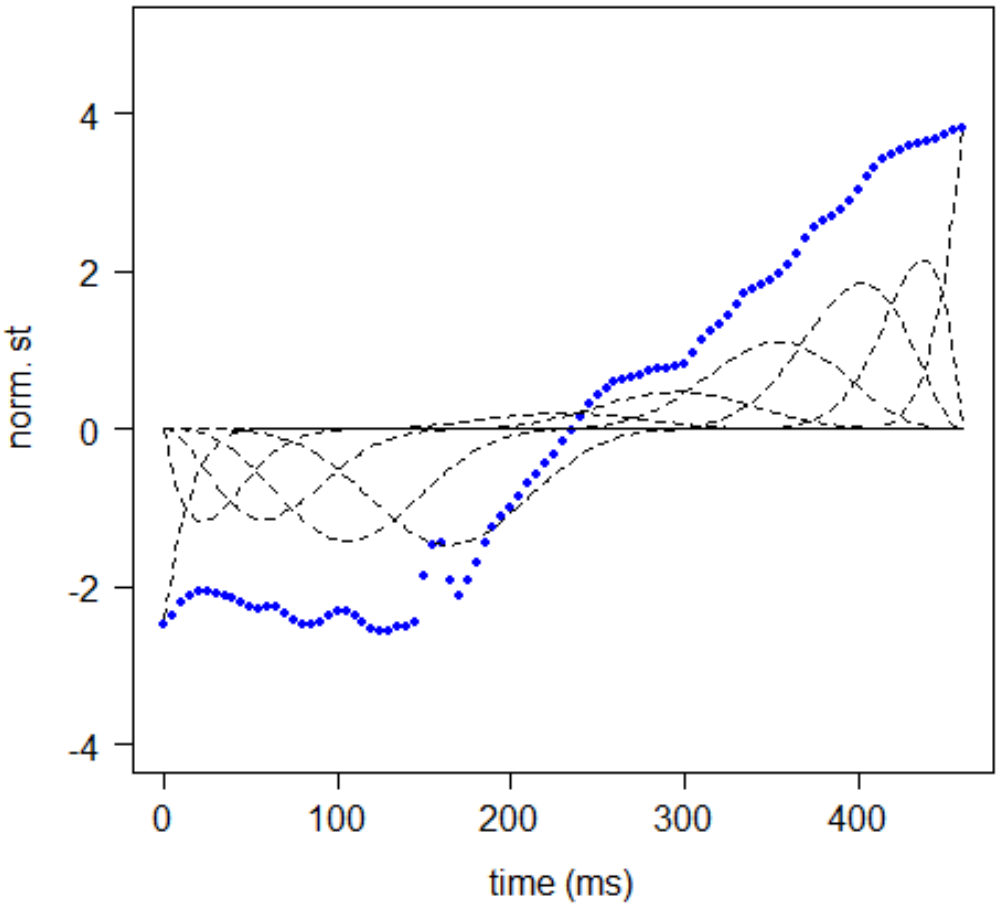
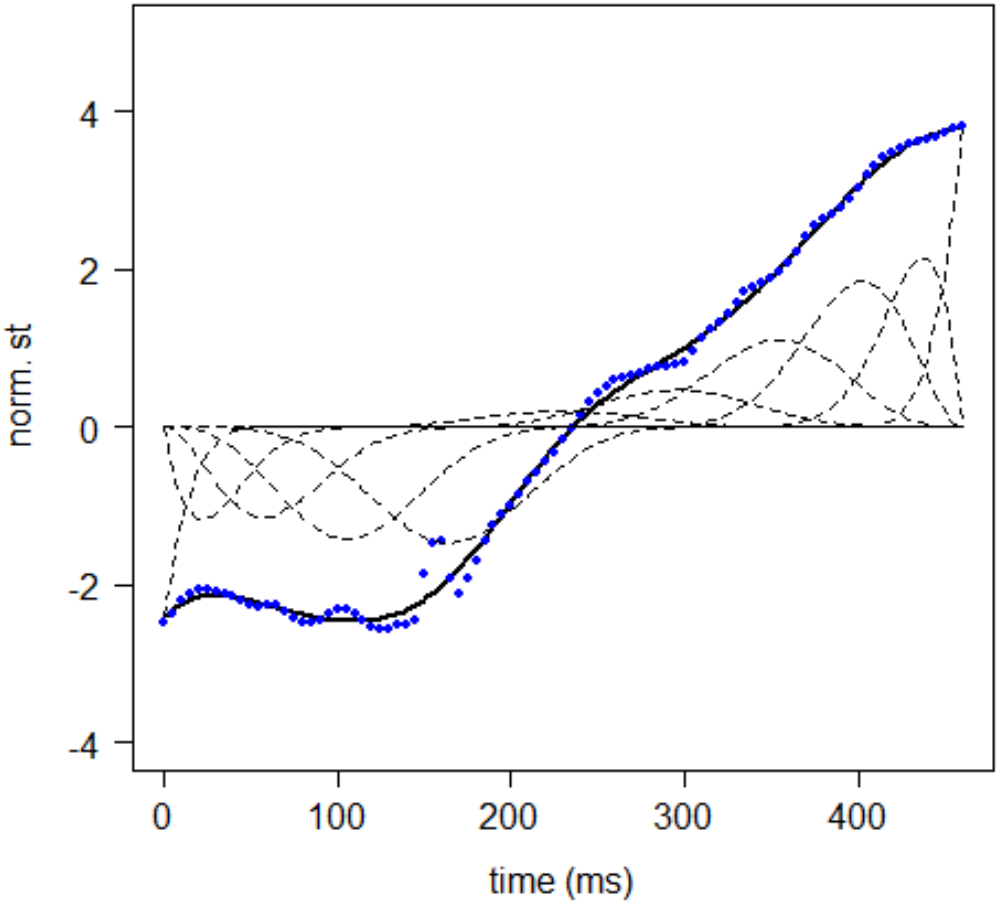
Interpolation with B-splines



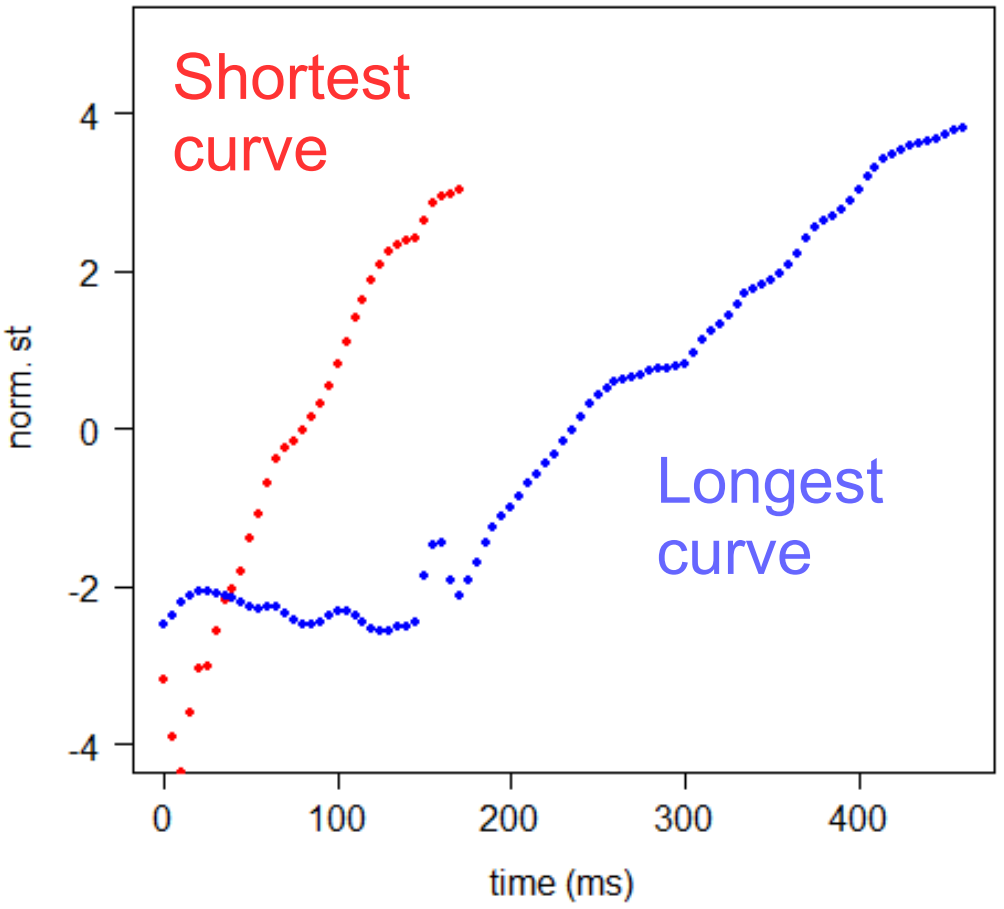
Interpolation with B-splines



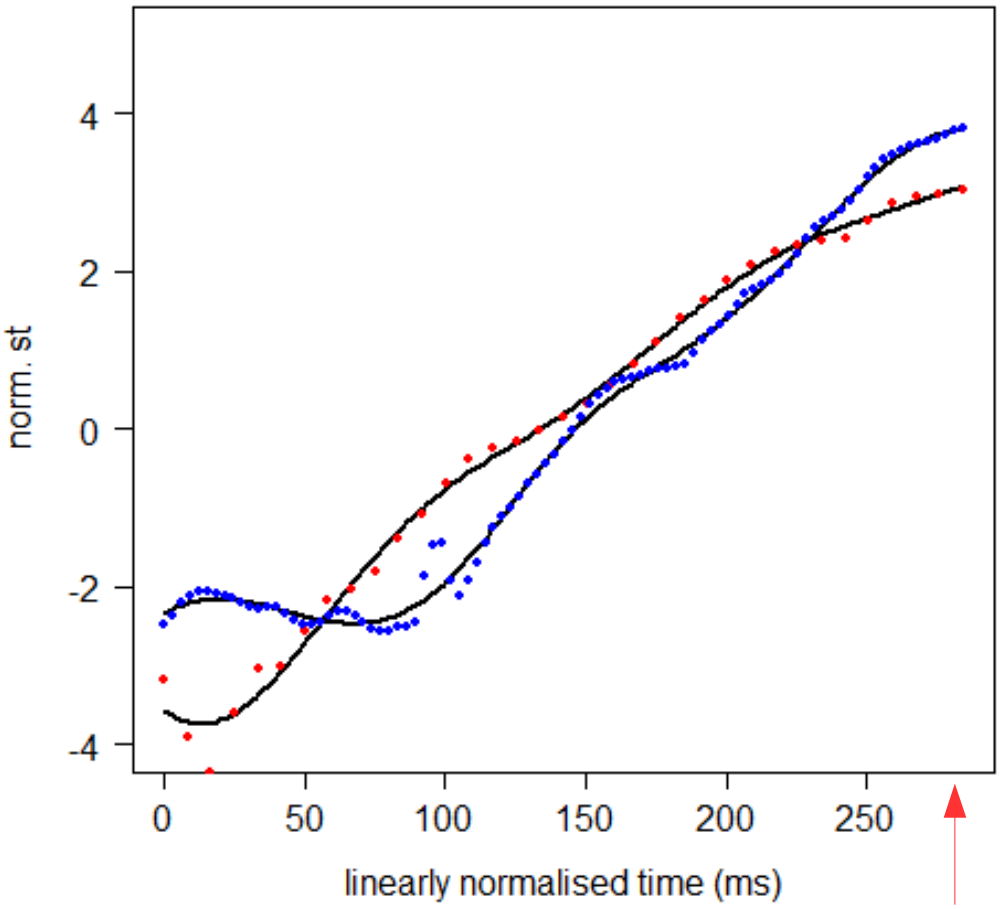
Interpolation with B-splines



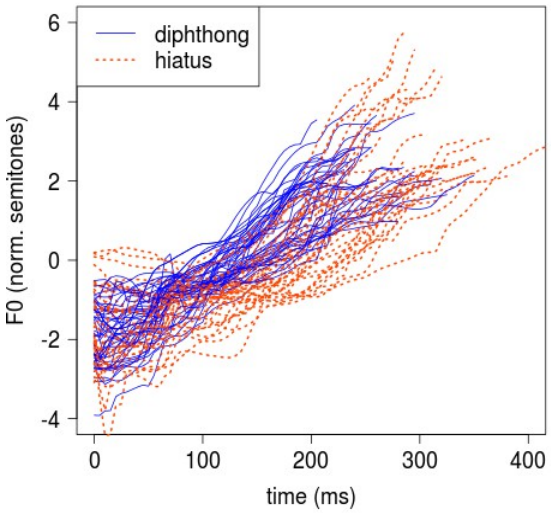
Different durations



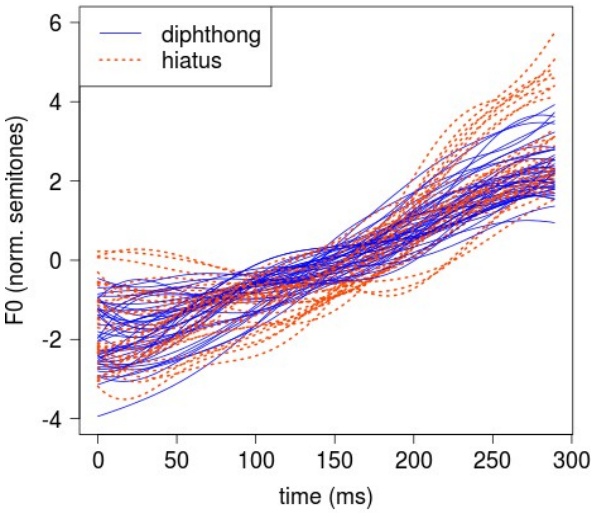
Linear time normalisation



Linear time normalisation



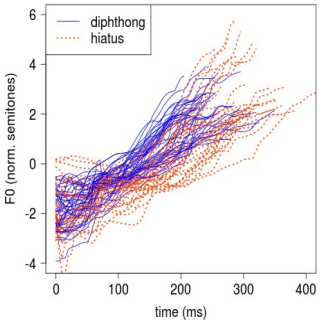
Interpolate on a common time interval



- We must use a common time interval
- This induces a linear time normalisation
- Durations have to be reintroduced at the end of the analysis

Road map

CURVES



Interpolate on a common time interval

- Data driven

Dimensionality reduction tool

- Few parameters
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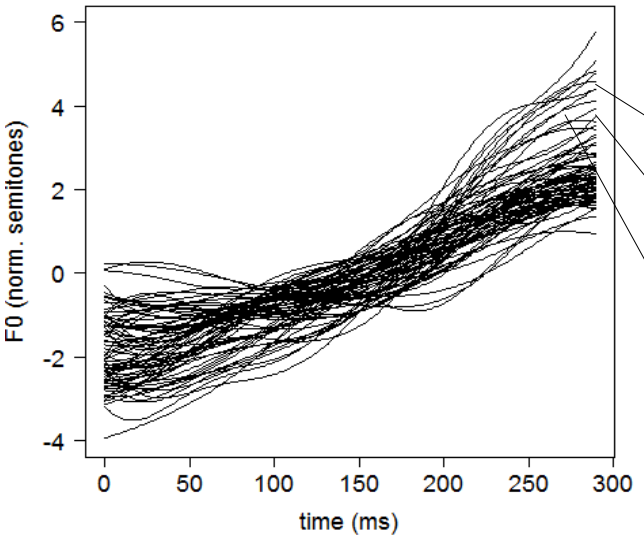
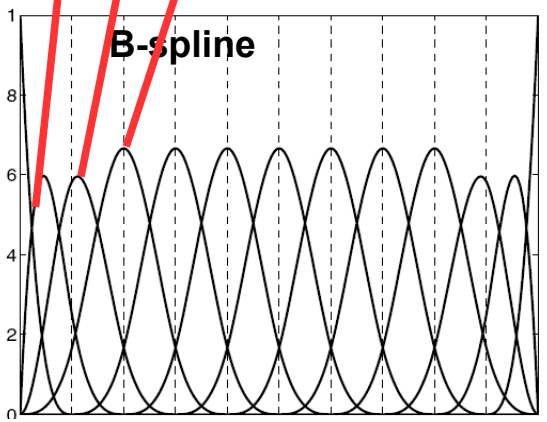
Principal Component Analysis



c1	c2	c3	...
...	...	...	...
...	...	...	...
...	...	...	...

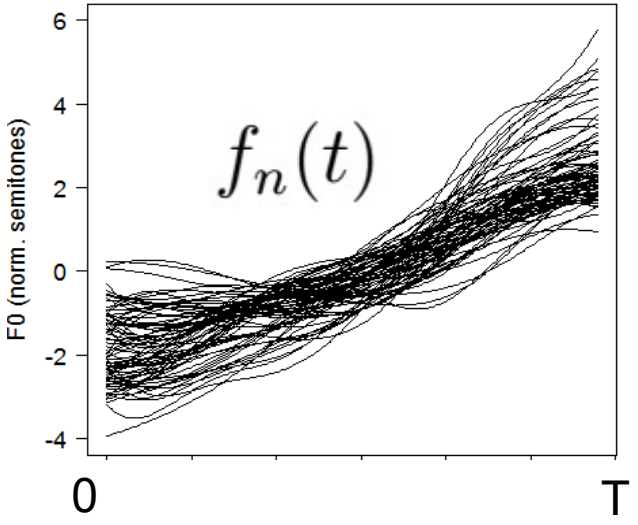


PCA

PCA limitations

- PCA does not use any explicit information related to the curve shapes or the B-splines shapes
- e.g. the sequence of coefficients c1, c2,.. reflects time adjacency of polynomial components, i.e. overlapping 'hills'

Functional PCA



$$\max\left\{var_n\left(\int_0^T PC1(t)f_n(t)dt\right)\right\}$$

$$\text{subject to}\int_0^T PC1^2(t)=1$$

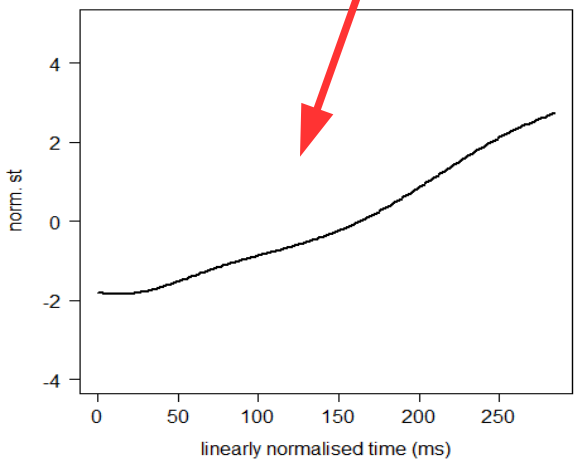
- FPCA definition uses the input curves $f_n(t)$
- FPCA is independent of the B-splines used to smooth $f_n(t)$

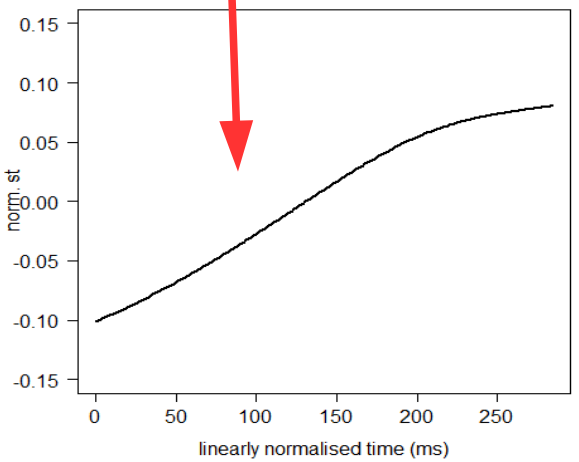
Functional PCs

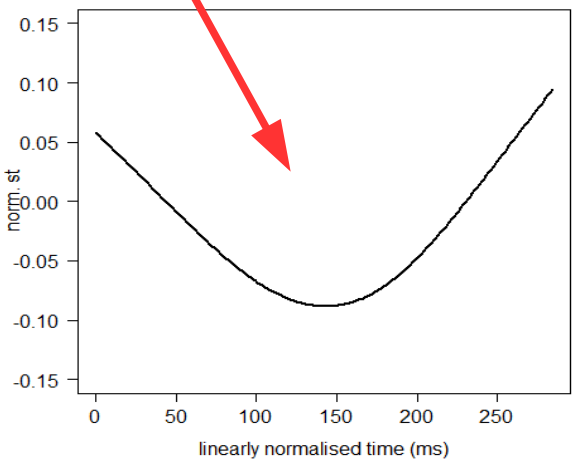
$$f(t) \approx \mu(t) + s_1 \cdot PC1(t) + s_2 \cdot PC2(t) + \cdots$$

PC1 score

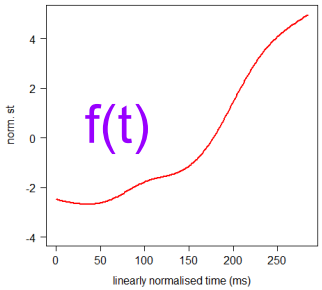
PC2 score







Curve reconstruction



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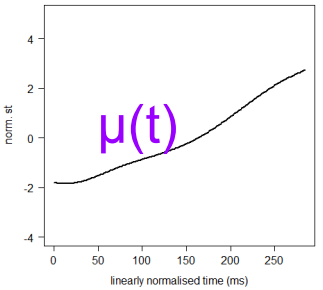
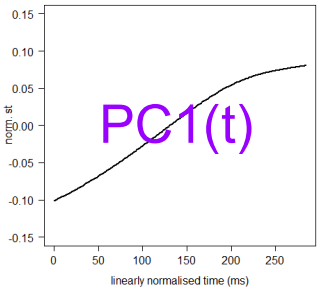
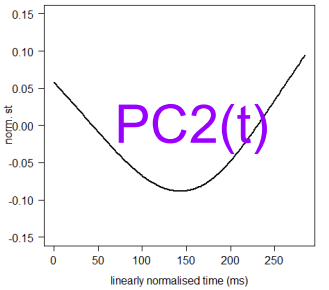
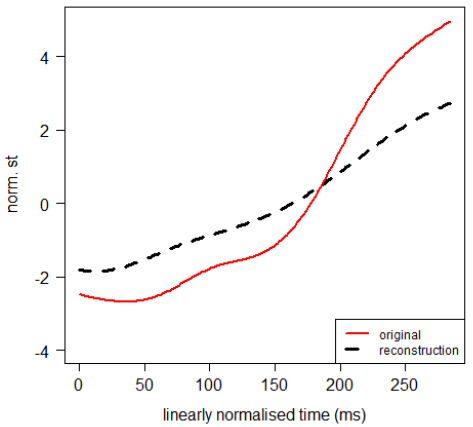


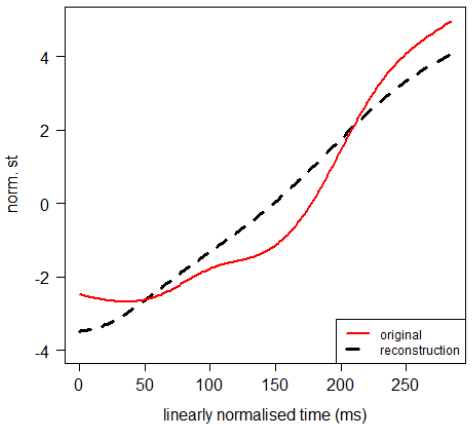
+ 16.5 *

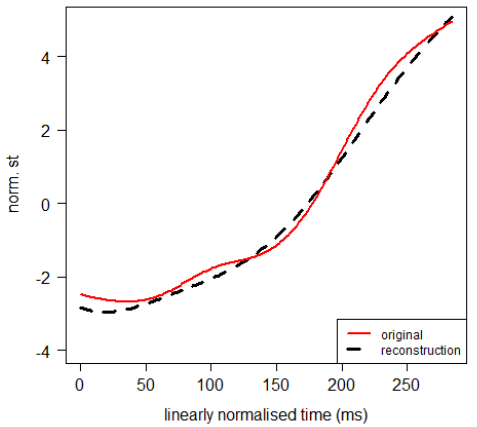


+ 10.8 *

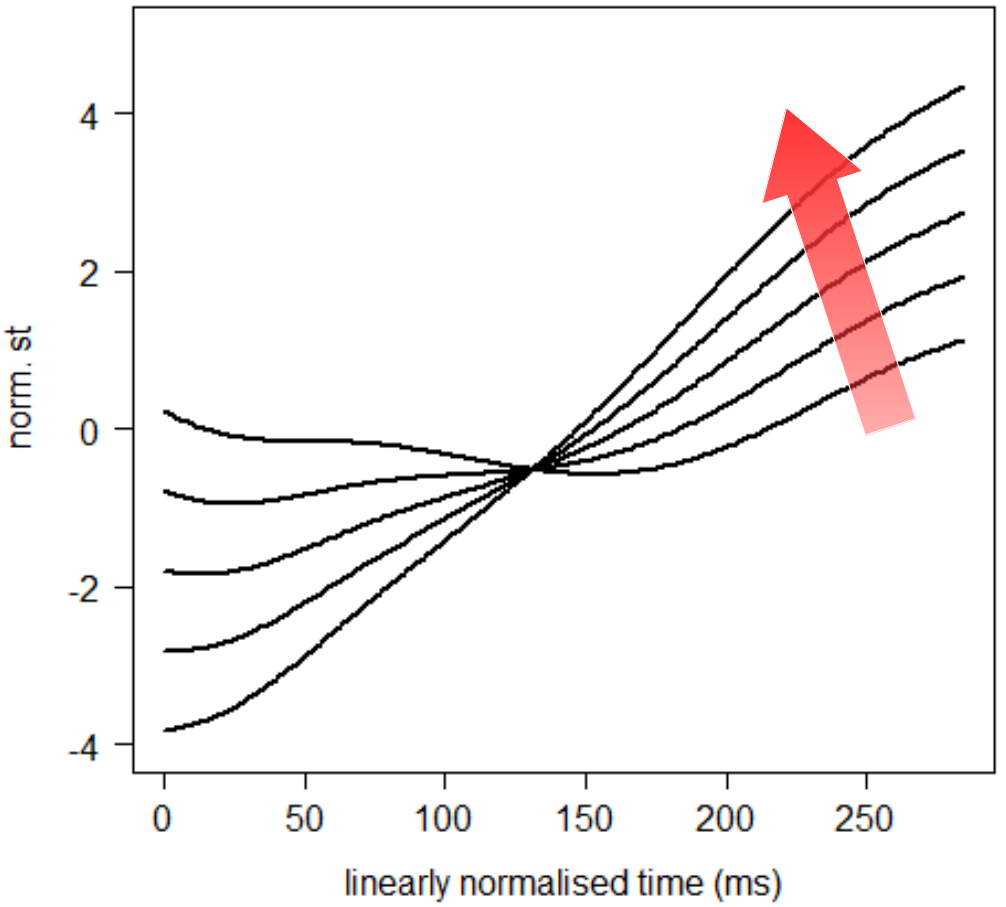






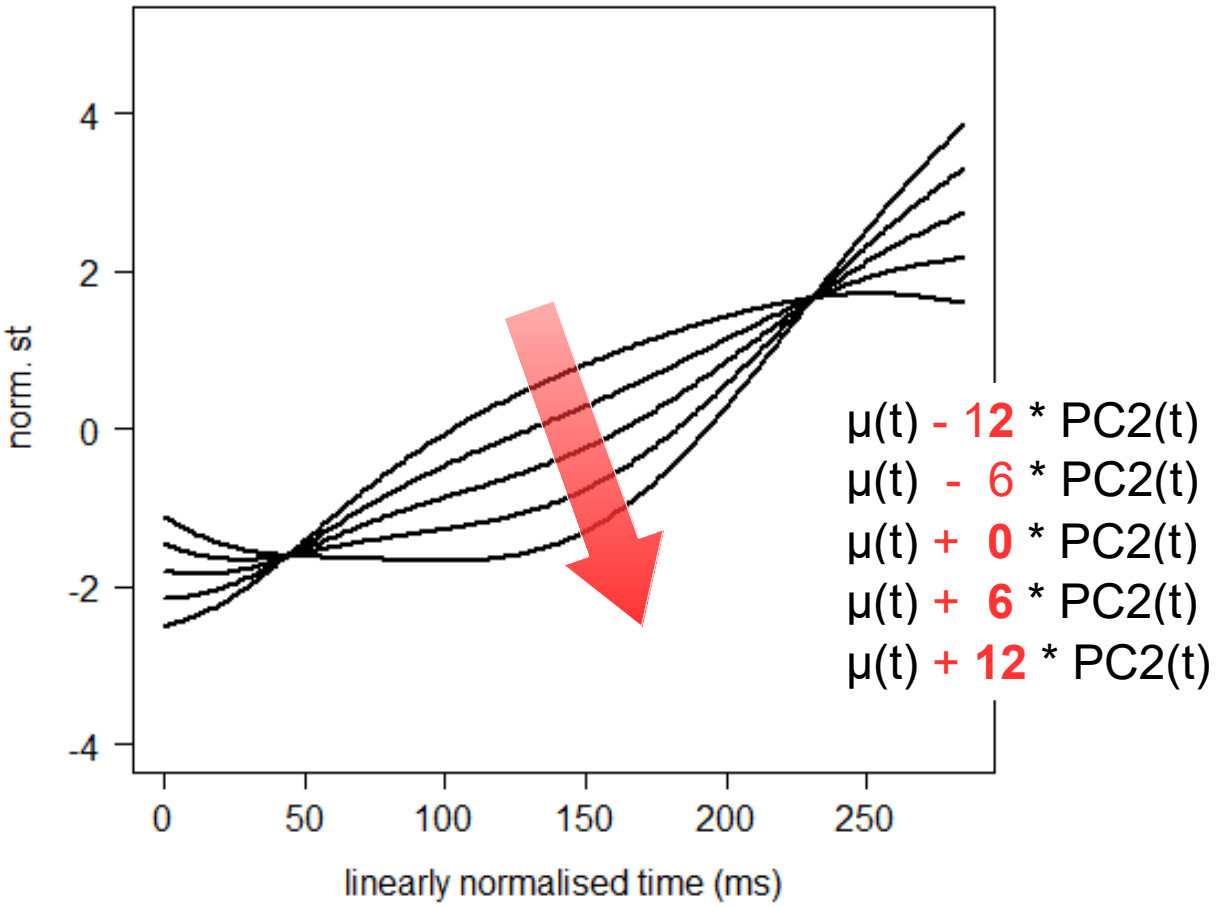


PC1 scores

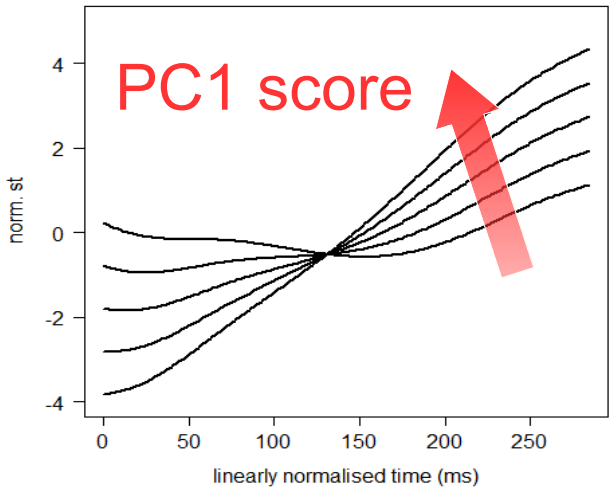


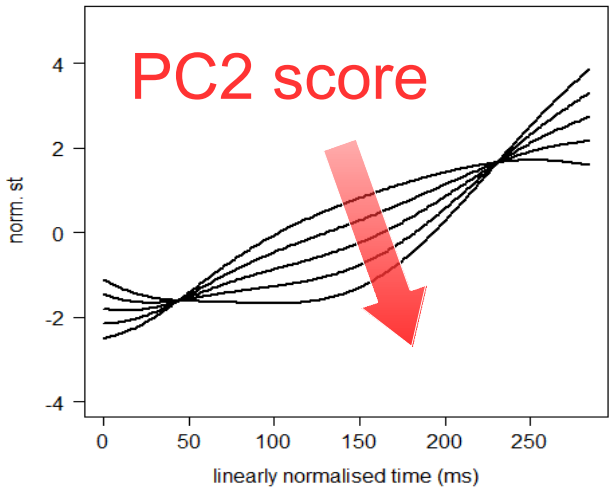
μ(t) + **20** * PC1(t)
μ(t) + **10** * PC1(t)
μ(t) + **0** * PC1(t)
μ(t) - **10** * PC1(t)
μ(t) - **20** * PC1(t)

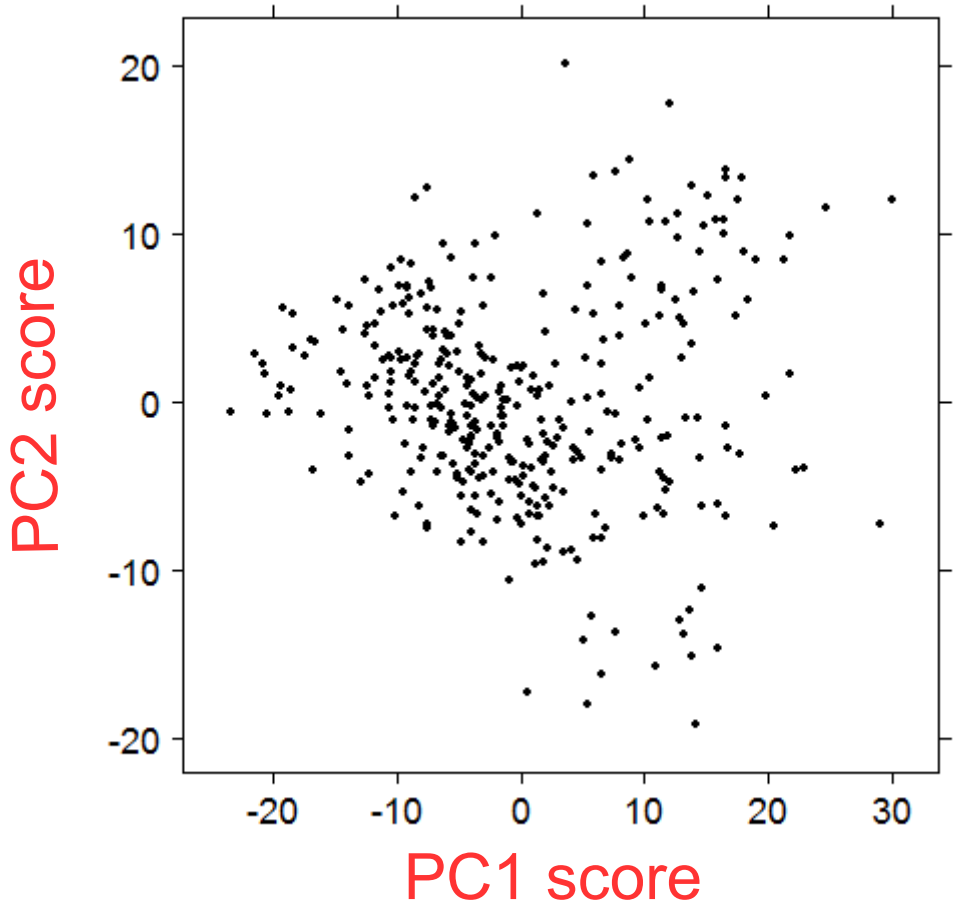
PC2 scores


4
2
0
-2
-4
norm. st
0
50
100
150
200
250
linearly normalised time (ms)
μ(t) - 12 * PC2(t)
μ(t) - 6 * PC2(t)
μ(t) + 0 * PC2(t)
μ(t) + 6 * PC2(t)
μ(t) + 12 * PC2(t)

Curve parametrisation







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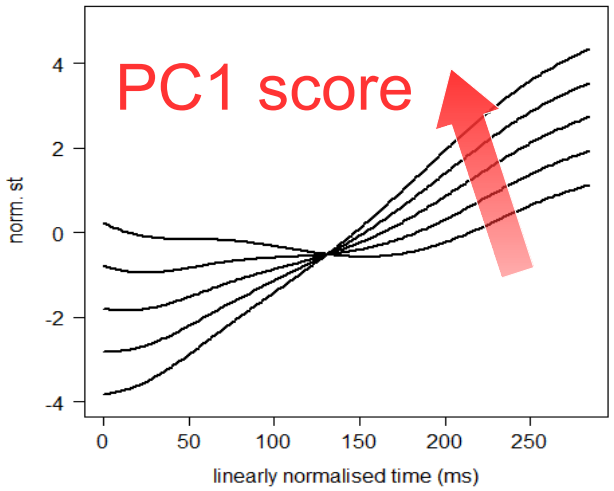
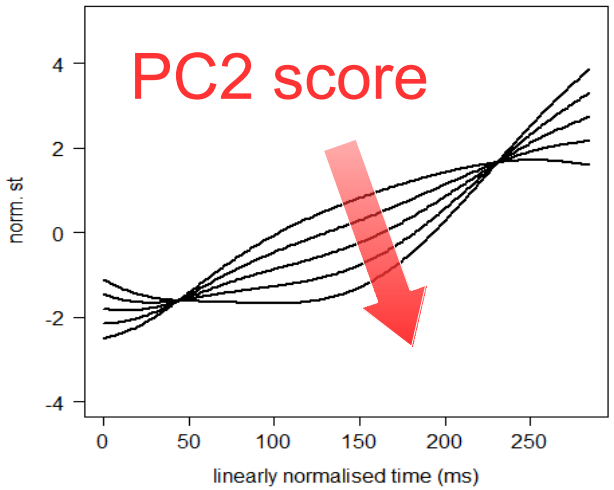
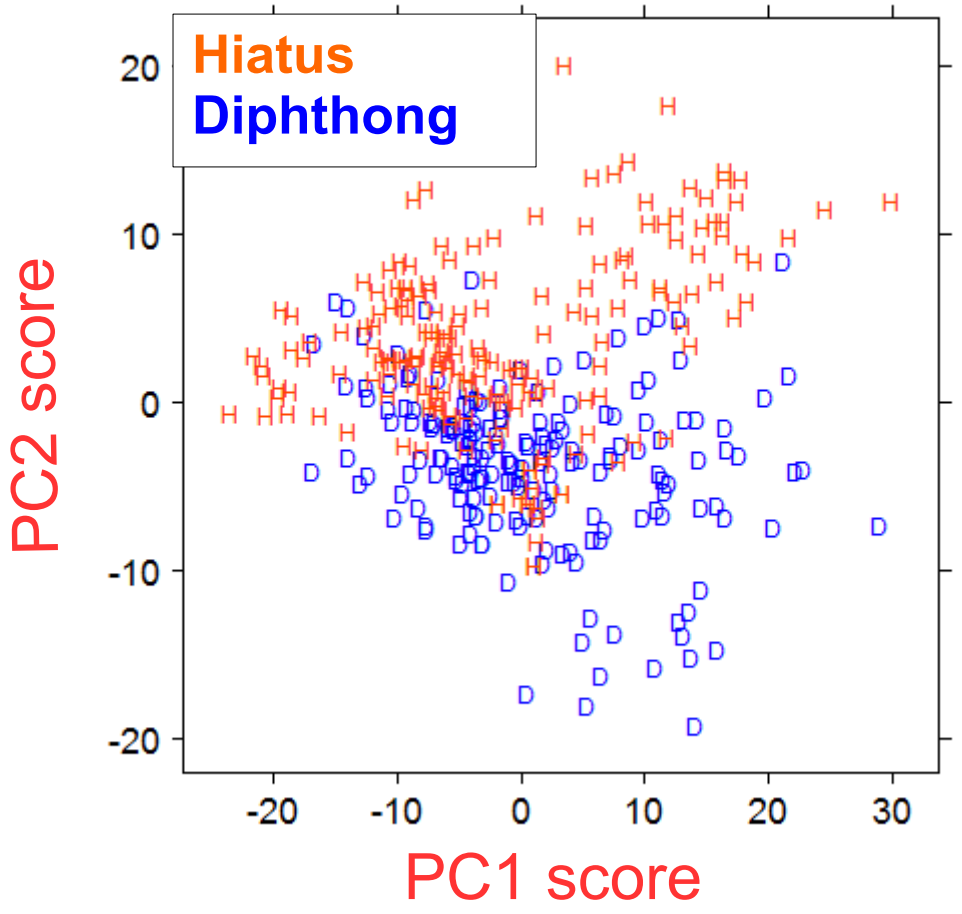
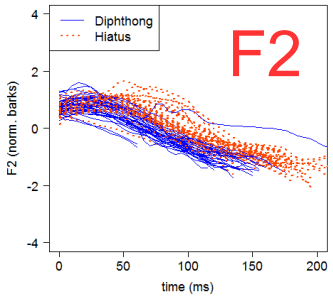


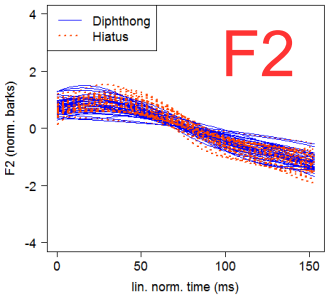


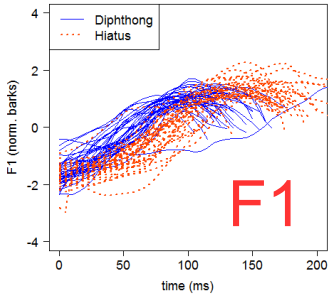


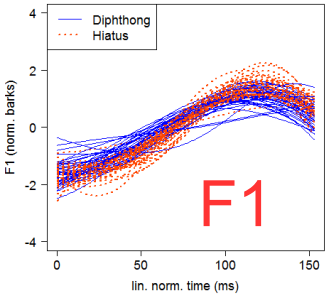
Formants

2D CURVES









FPCA

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NUMBERS

ANOVA

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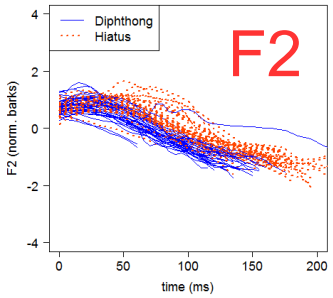
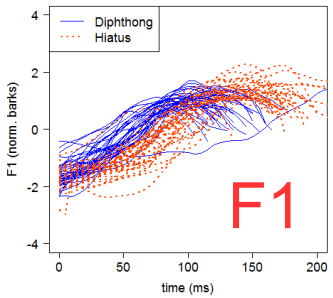
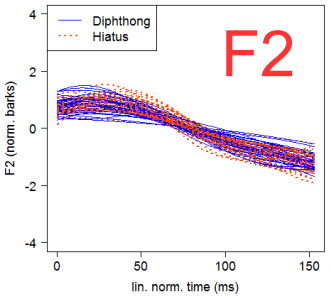
LMER

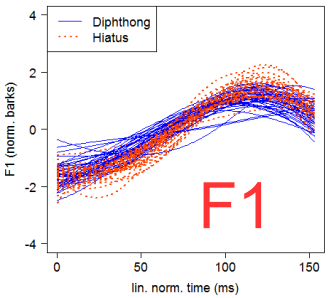
Formants

2D CURVES









2D
FPCA

NUMBERS

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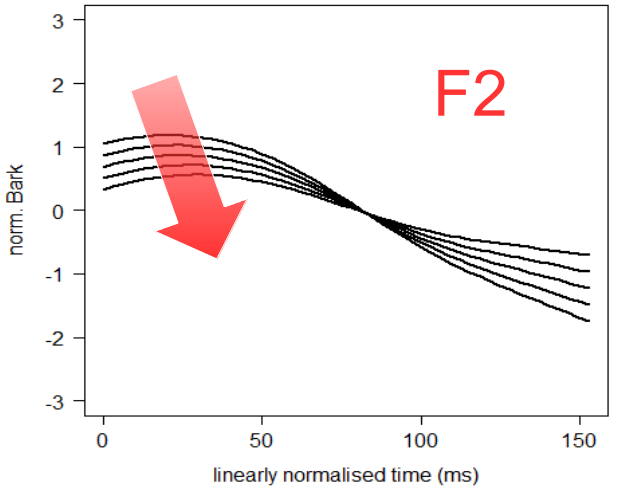
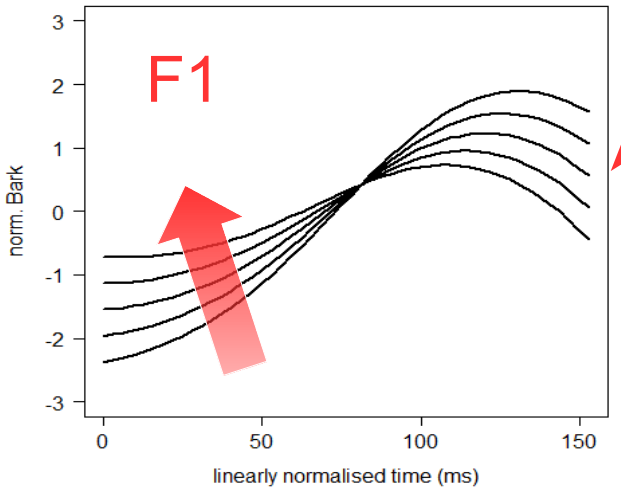
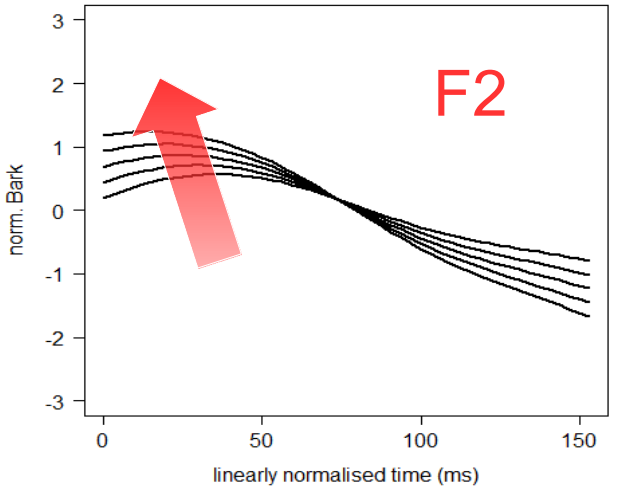
PC1 scores

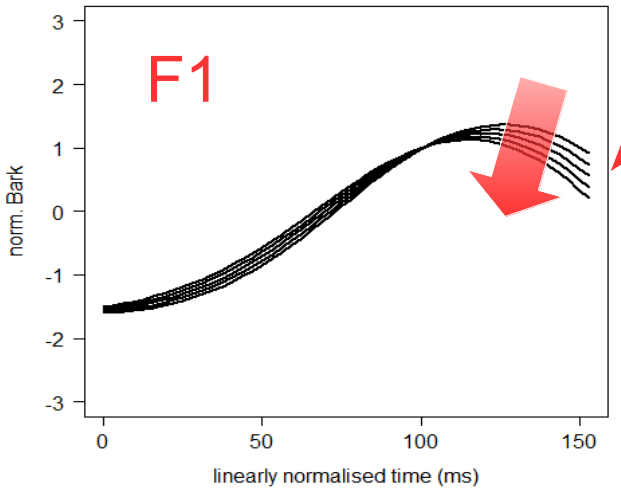




μ(t) + **8** * PC1(t)
μ(t) + **4** * PC1(t)
μ(t) + **0** * PC1(t)
μ(t) - **4** * PC1(t)
μ(t) - **8** * PC1(t)

PC2 scores

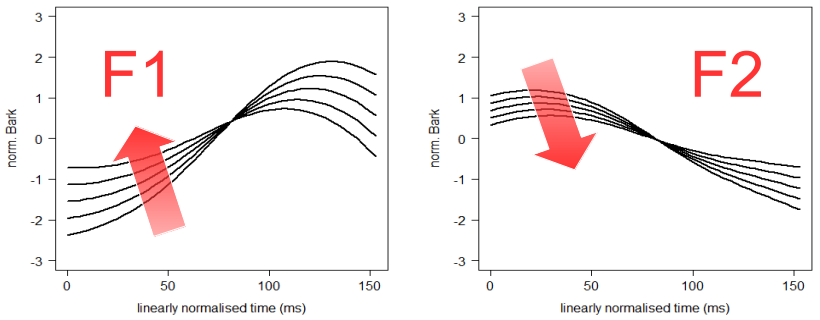




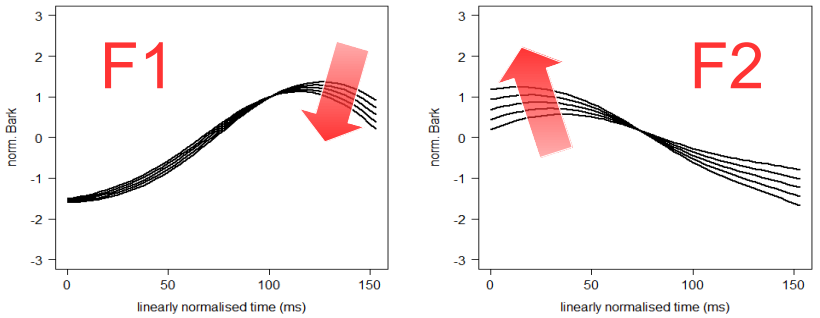
μ(t) + **4** * PC1(t)
μ(t) + **2** * PC1(t)
μ(t) + **0** * PC1(t)
μ(t) - **2** * PC1(t)
μ(t) - **4** * PC1(t)

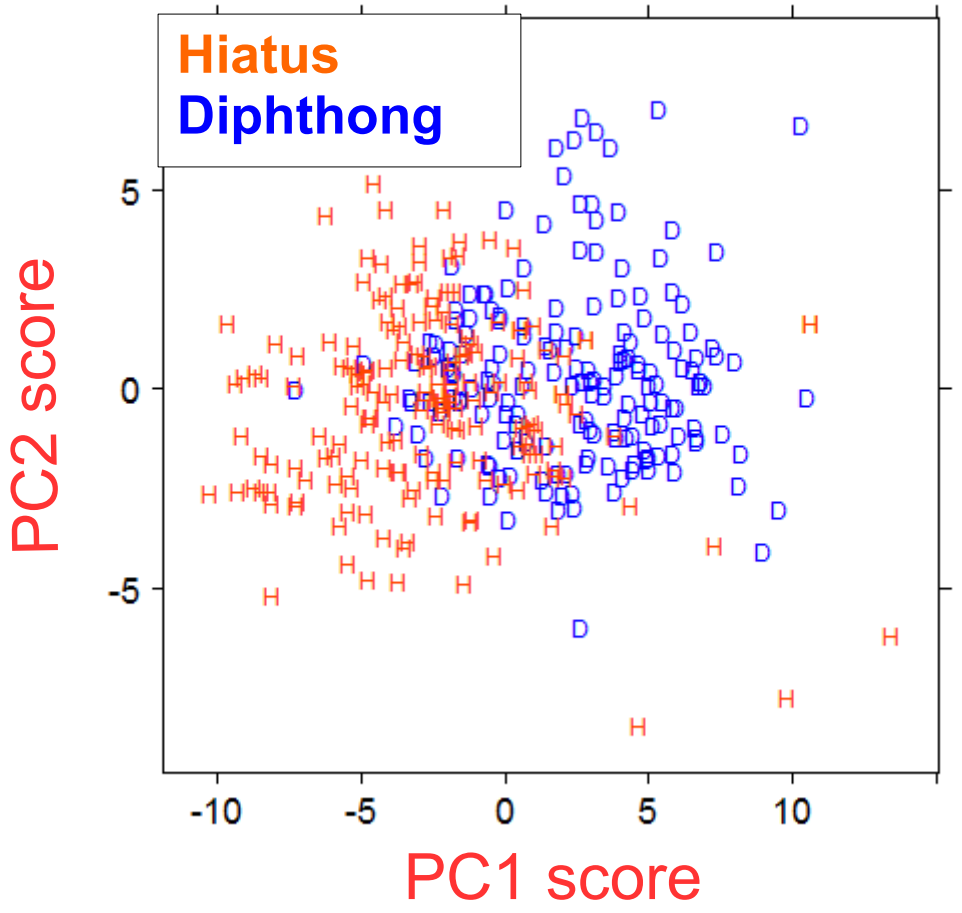
2D curve parametrisation

PC1 score

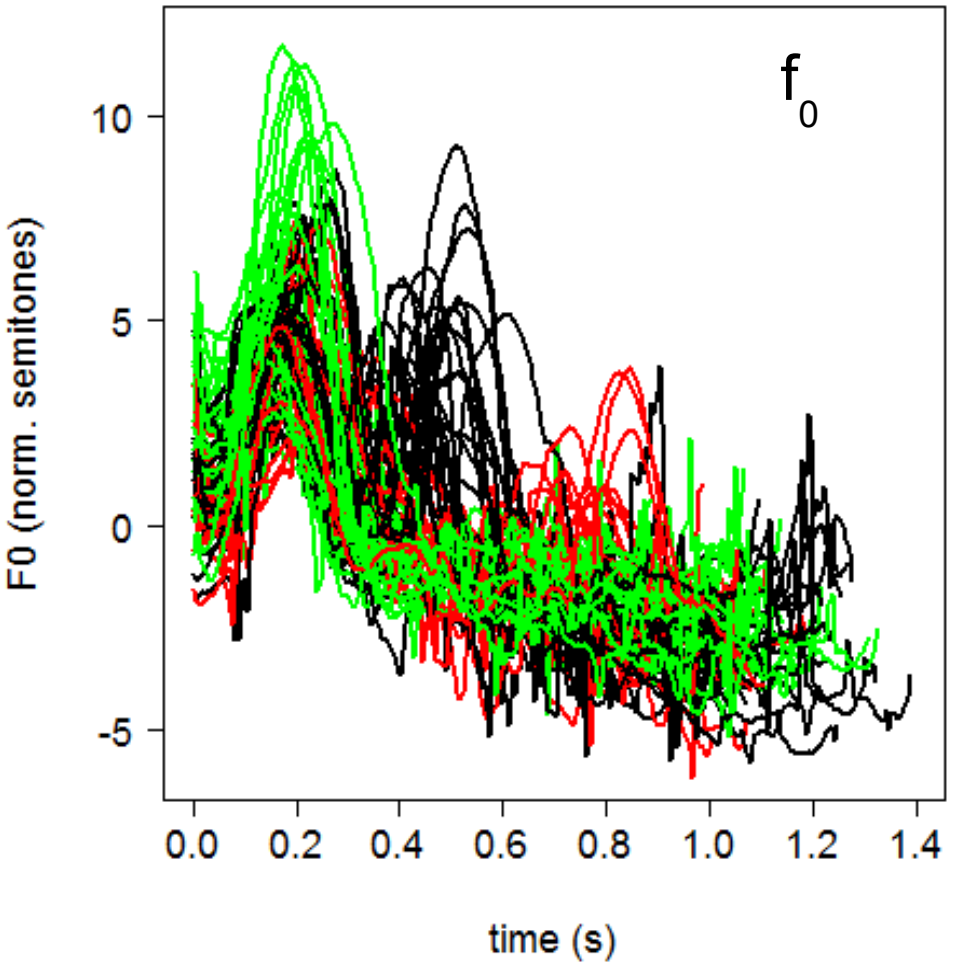


PC2 score





Many segments



- Narrow focus in Neapolitan Italian
- Focus on

  Subject, **Verb** or **Prop. Phrase**

  Danilo vola da Roma

  (*Danilo flies from Rome*)
- 8 CV syllables

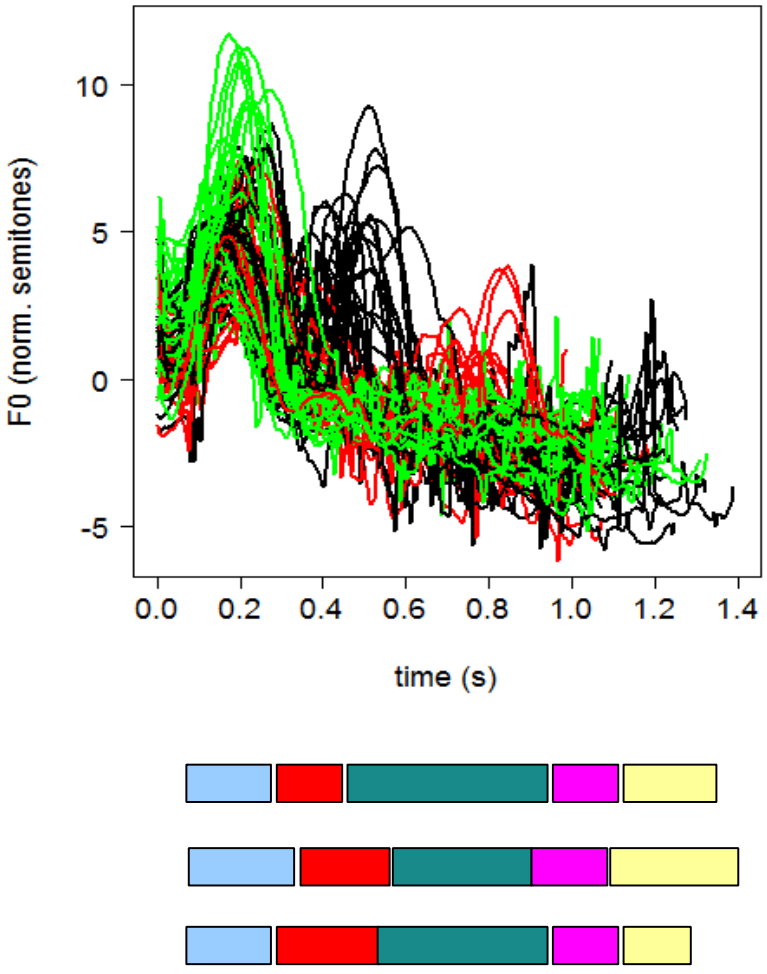
  first C was excluded (too short)

  VCVCV CVCV CV CVCV

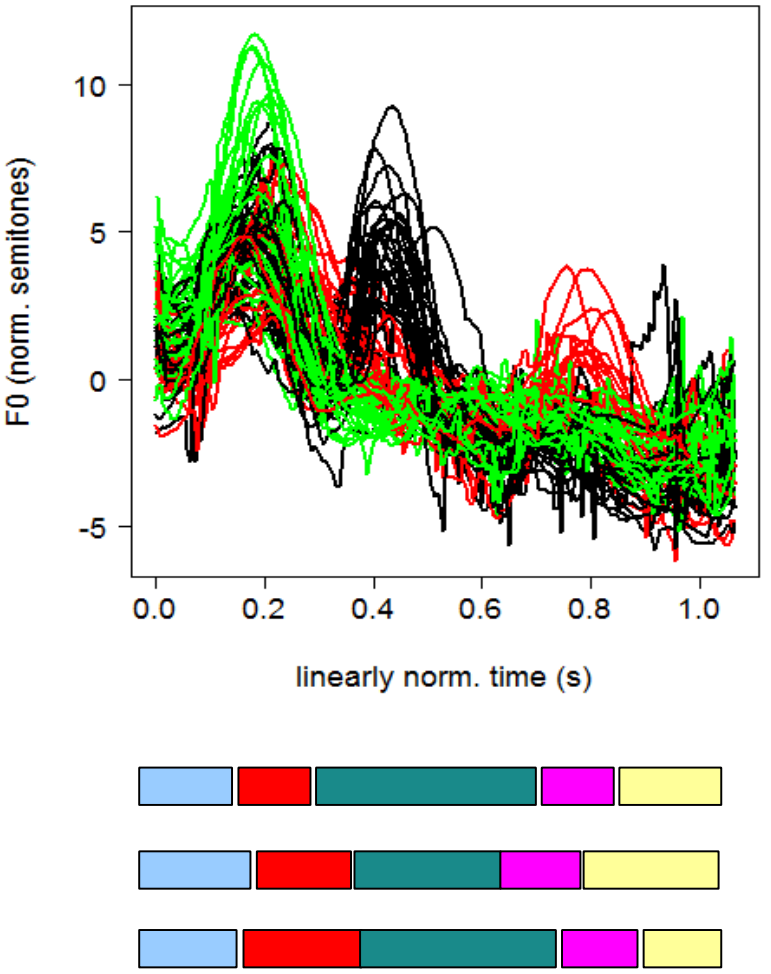
… 15 segments!

Linear time normalisation

BEFORE

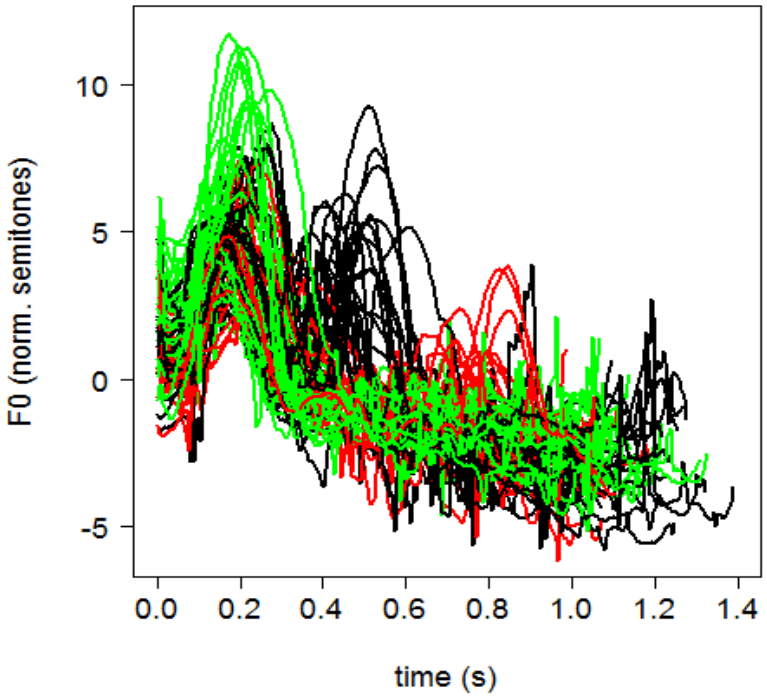


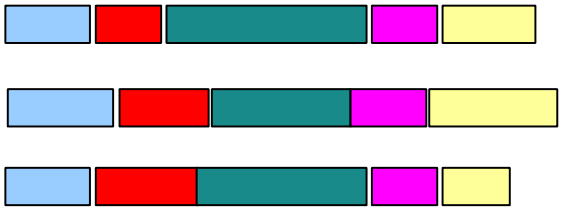
AFTER



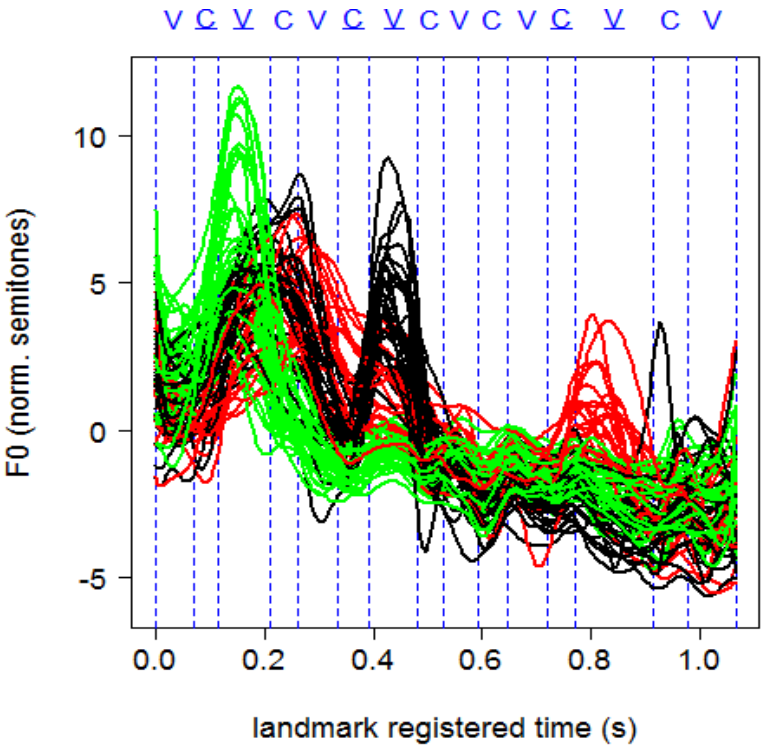
Landmark registration

BEFORE



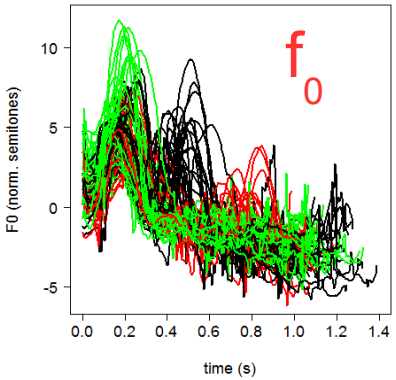


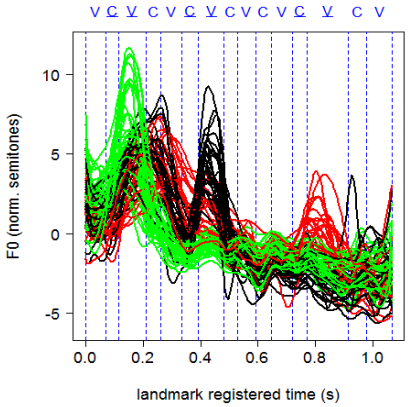
AFTER



Using landmark registration

CURVES





segment durations

d1	d2	...	d15
...	...	...	...
...	...	...	...
...	...	...	...

FPCA

PCA

NUMBERS

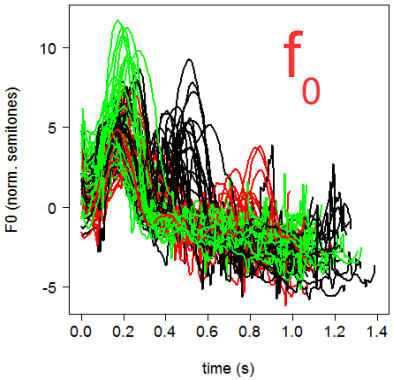
ANOVA

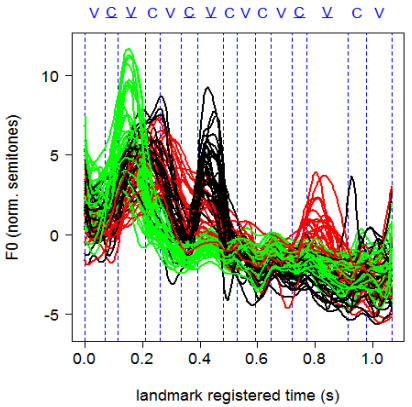
LM

LMER

Using landmark registration

CURVES





segment durations

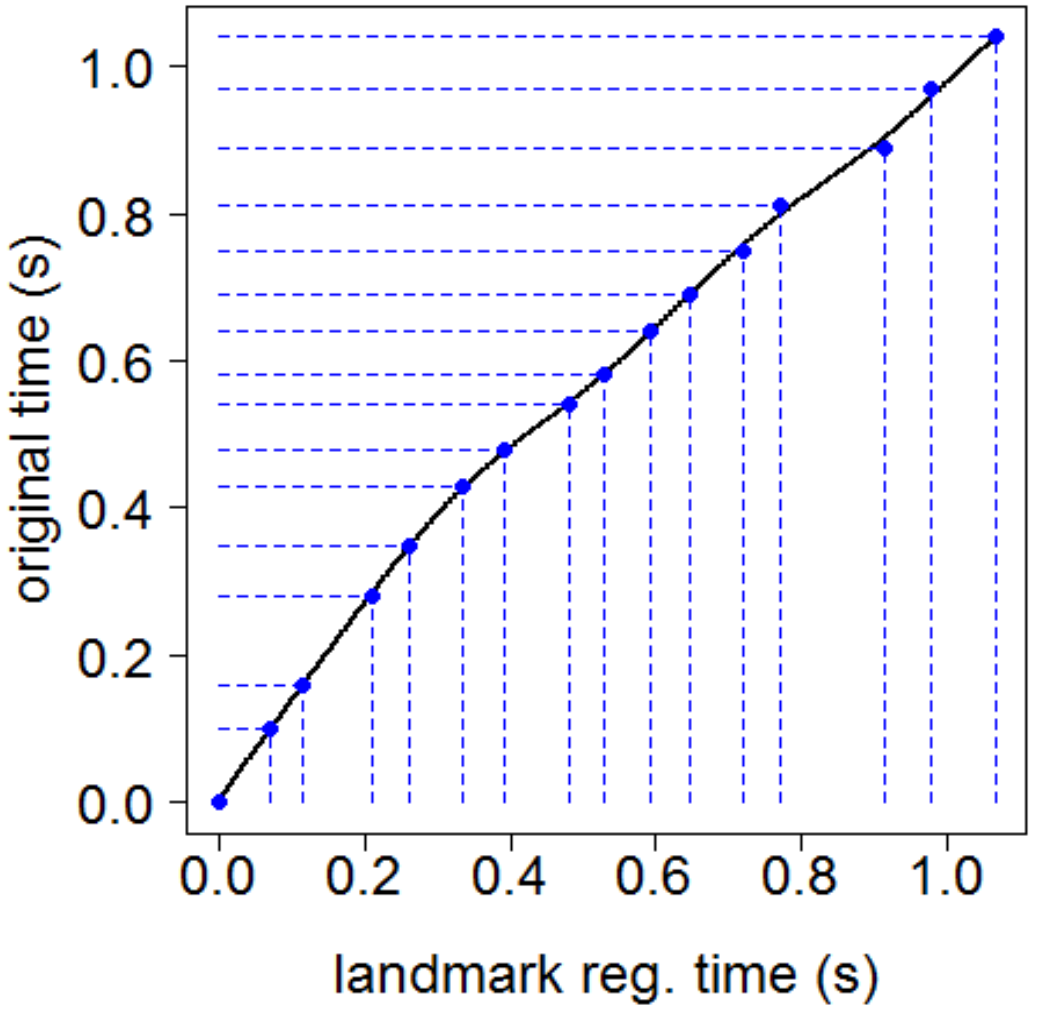
d1	d2	...	d15
...	...	...	...
...	...	...	...
...	...	...	...

FPCA

NUMBERS

- ANOVA
- LM
- LMER

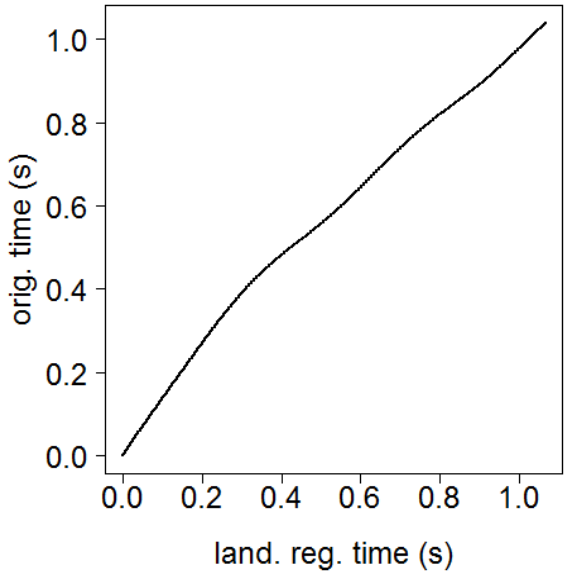
Inside landmark registration

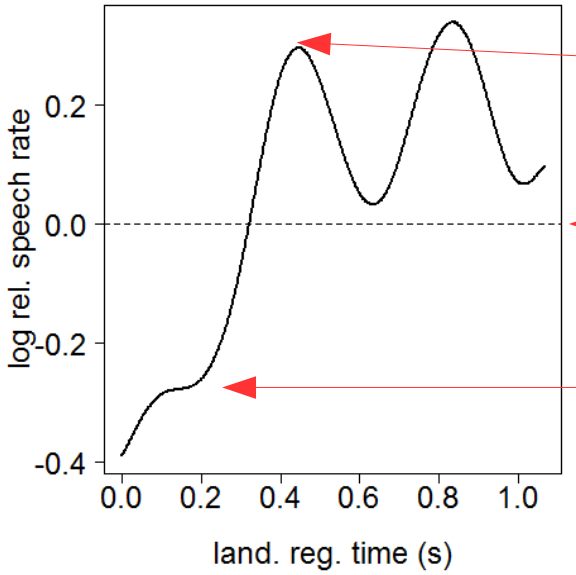


h(t)

Relative log rate

h(t) ⟺ - log dh(t)/dt





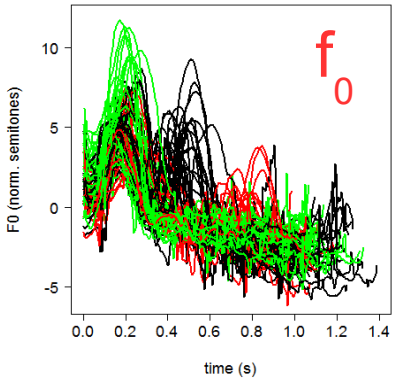
+ 0.25 → duration / 1.28

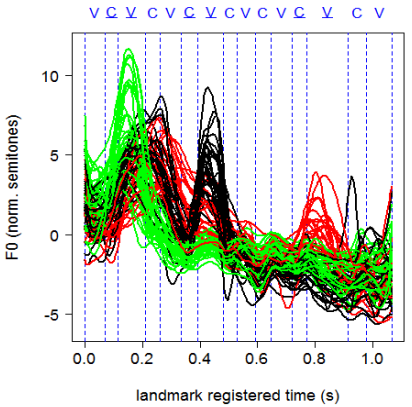
0 → same duration

- 0.25 → duration * 1.28

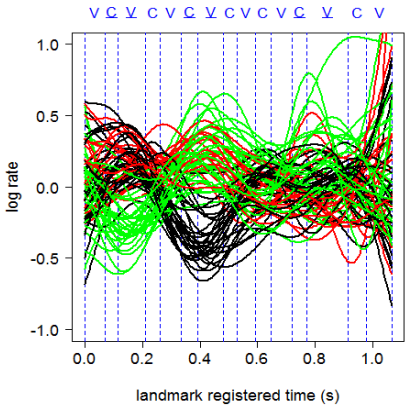
Using log rates

CURVES





log rates

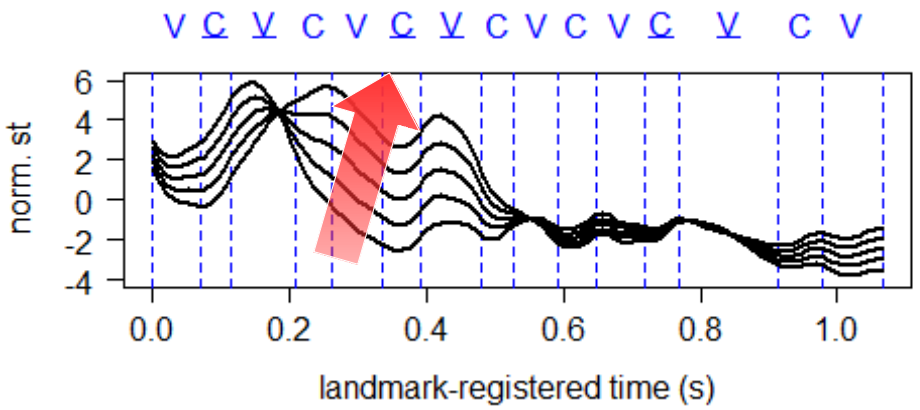


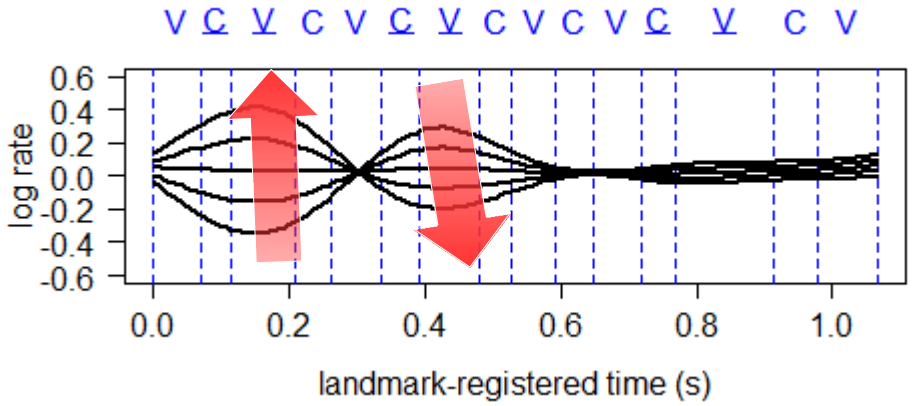
2D FPCA

NUMBERS

- ANOVA
- LM
- LMER

PC1 scores



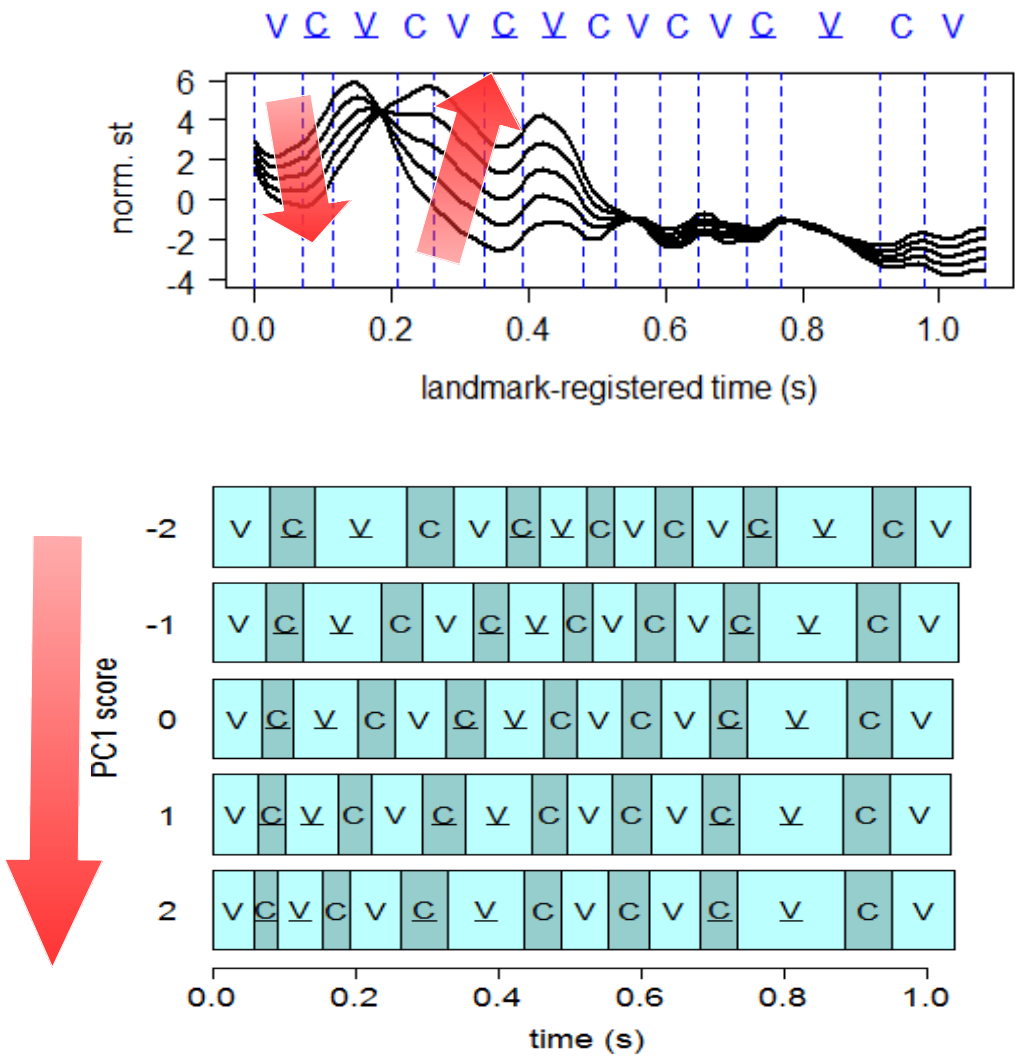


f_0

log rates

$\mu(t) + 2 * PC1(t)$
$\mu(t) + 1 * PC1(t)$
$\mu(t) + 0 * PC1(t)$
$\mu(t) - 1 * PC1(t)$
$\mu(t) - 2 * PC1(t)$

PC1 scores

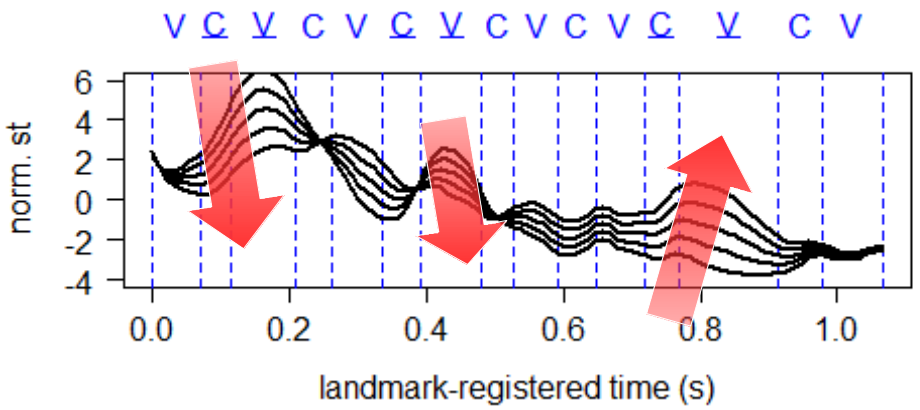


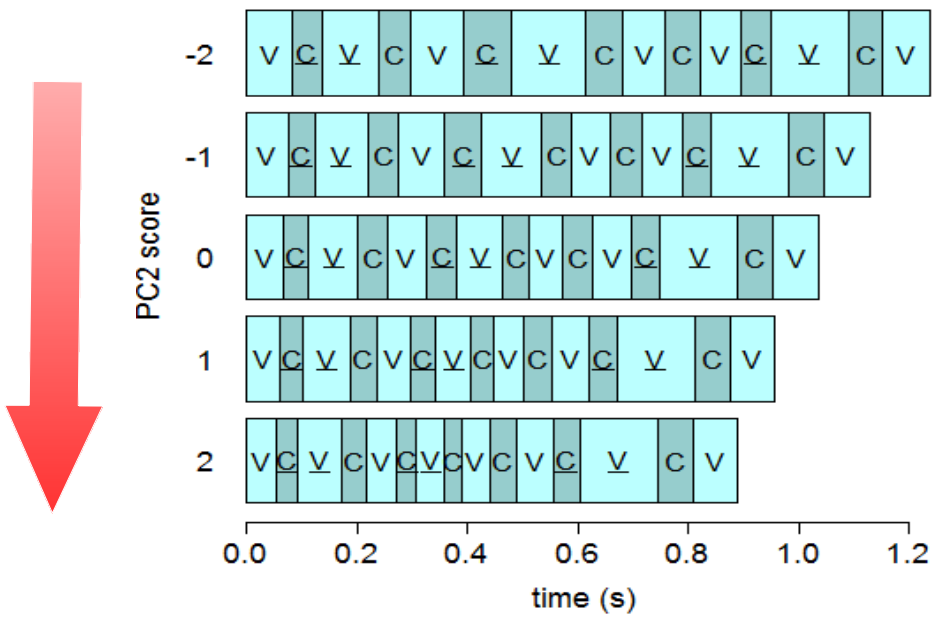
f_0

$\mu(t) + 2 * PC1(t)$
$\mu(t) + 1 * PC1(t)$
$\mu(t) + 0 * PC1(t)$
$\mu(t) - 1 * PC1(t)$
$\mu(t) - 2 * PC1(t)$

segment durations

PC2 scores





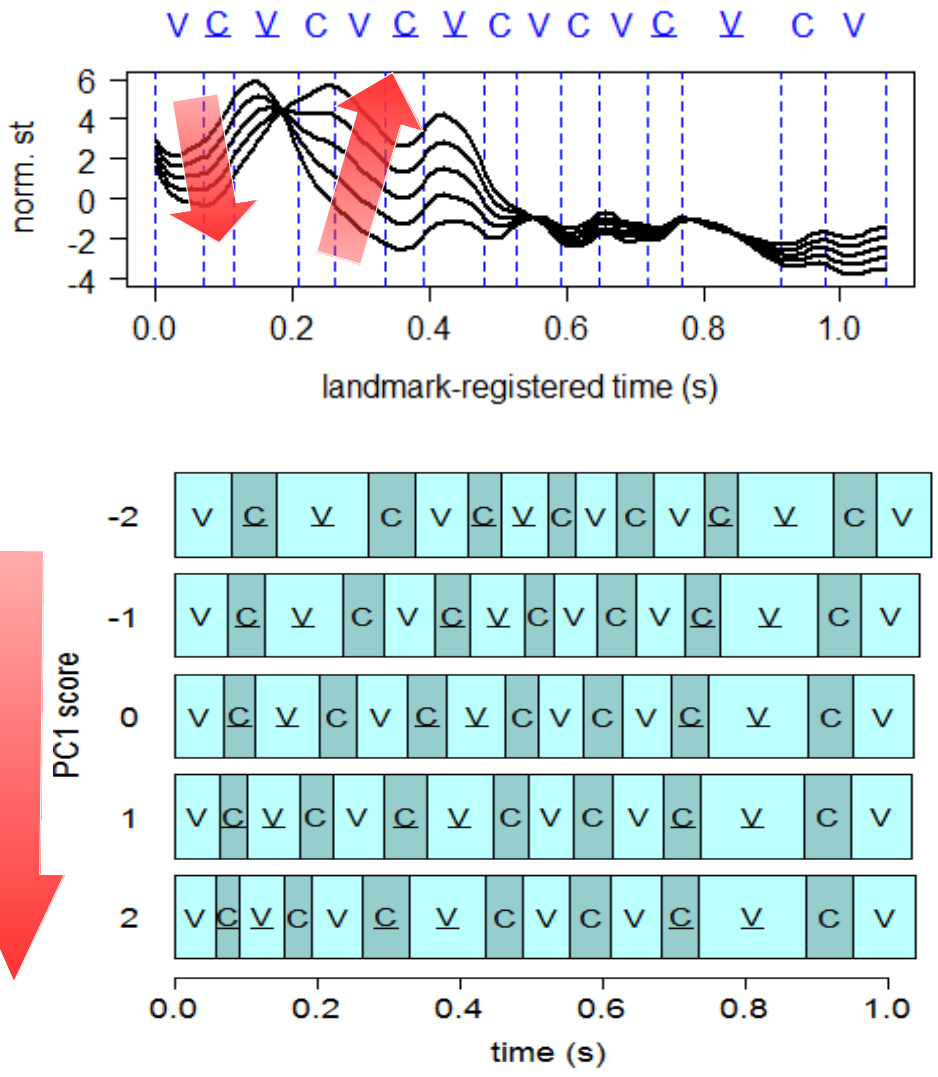
f_0

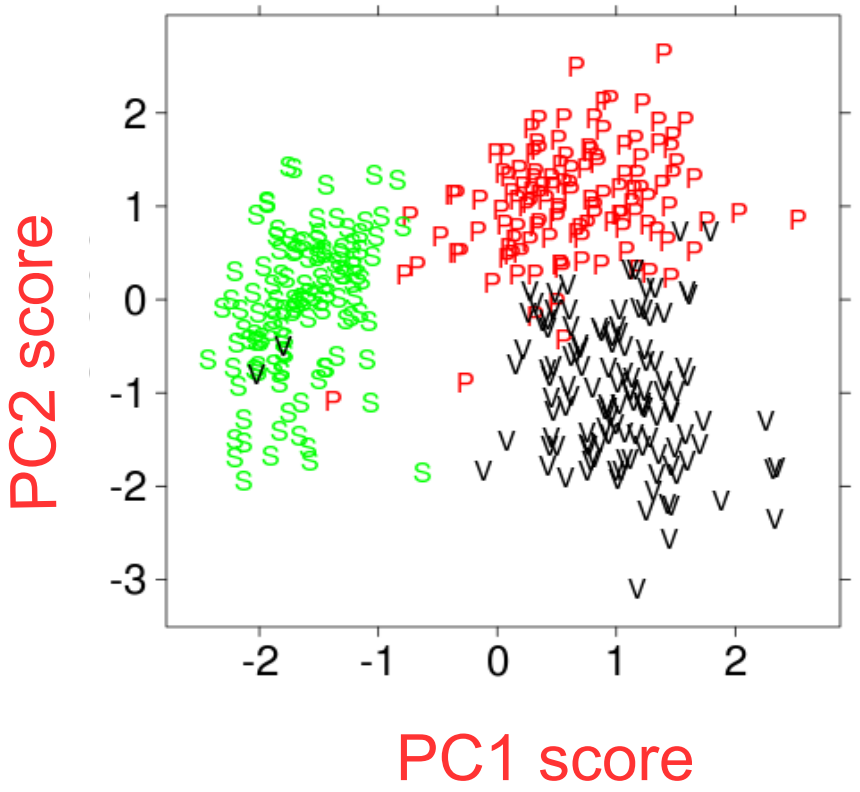
$\mu(t) + 2 * PC1(t)$
$\mu(t) + 1 * PC1(t)$
$\mu(t) + 0 * PC1(t)$
$\mu(t) - 1 * PC1(t)$
$\mu(t) - 2 * PC1(t)$

segment durations

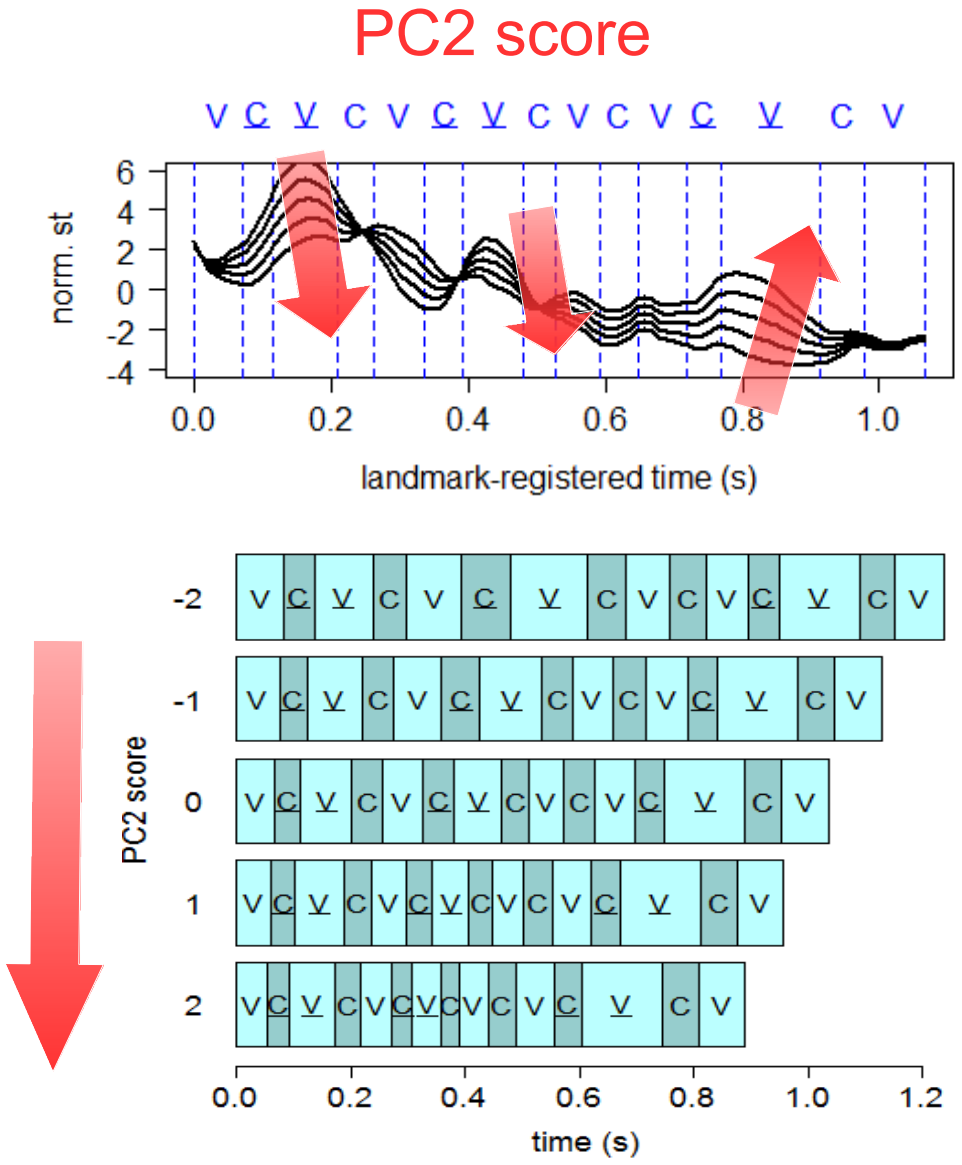
multi-segment curve parametrisation

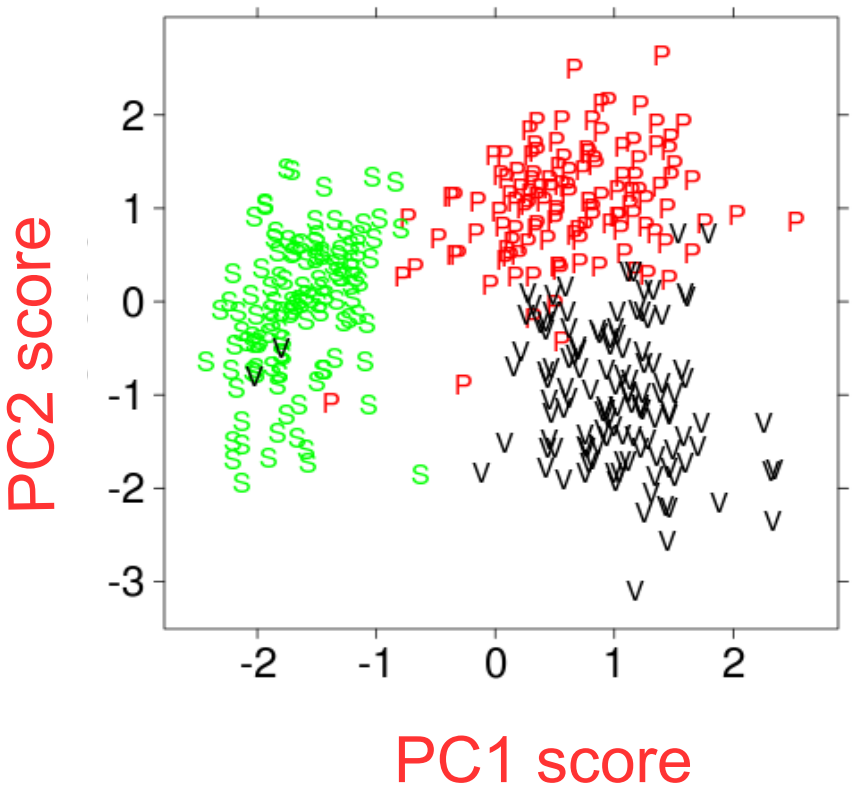
PC1 score





multi-segment curve parametrisation





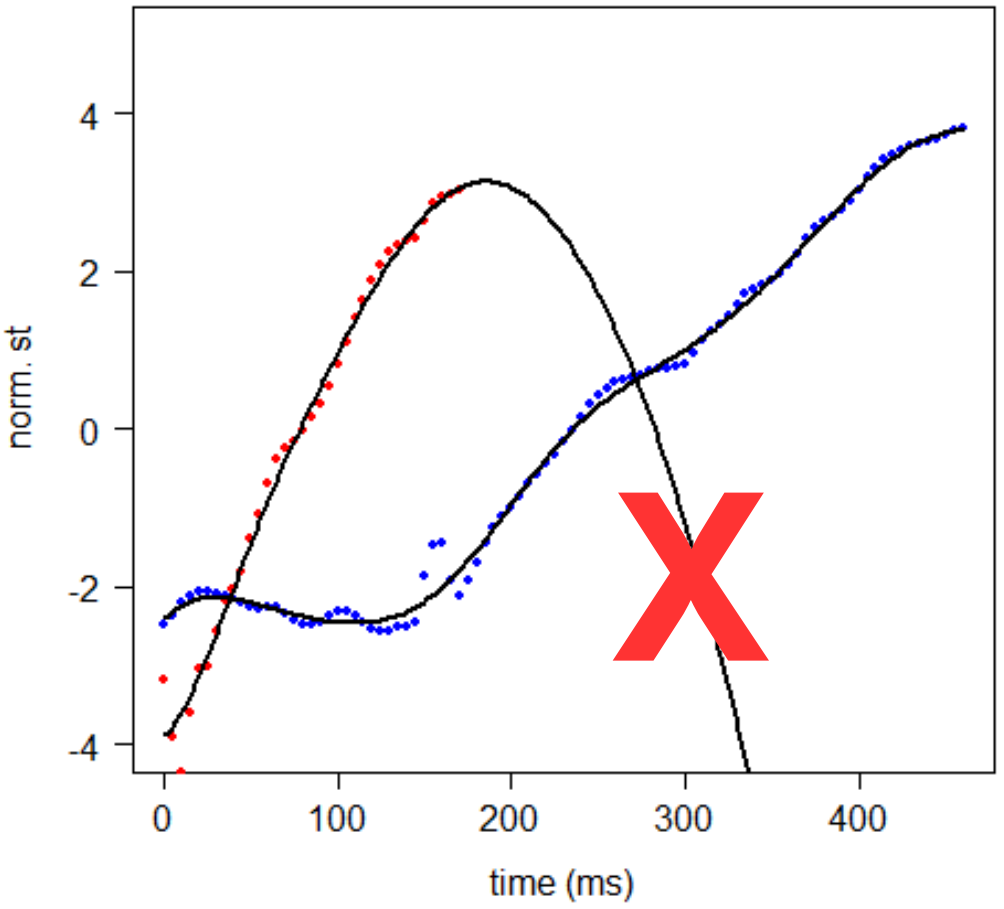
Try this at home?

- R package fda
- my website: http://lands.let.ru.nl/FDA/

  where you find papers, R scripts, tutorials, etc.
- There is more!
  - FPCA as re-synthesis tool for perceptual experiment
  - FPCA as map to save on manual (ToBI) annotation

appendix

Take longest duration



Take shortest duration